

Mu Alpha Theta National Convention 2004
 Mu Division Matrices and Vectors
 For all questions, NOTA means none of these answers.

1. Given: $A = \begin{bmatrix} 2a & -3b \\ 3 & 2 \end{bmatrix}$; $B = \begin{bmatrix} -3 & 1 \\ 5 & 2 \end{bmatrix}$; $C = \begin{bmatrix} 3 & 4 \\ 8 & 4 \end{bmatrix}$ with $A + B = C$. Find the product ab .

- a) -3 b) 0 c) 3 d) 5 e) NOTA

2. The solution to x of the following system is in the form of $\frac{a}{b}$. Find $a + b$.

$$\begin{aligned} 2x - 3y + z &= 3 \\ x - 2y - 2z &= 5 \\ 3x - y + 2z &= 8 \end{aligned}$$

- a) 88 b) 38 c) -11 d) -45 e) NOTA

3. The road to city A and city B is uphill for 5 miles, level for 4 miles, then downhill for 6 miles. Susan walked from city B to city A in 4 hours and later she walks back to city B in 3 hours and 52 minutes. At another time she walks half way from city A to city B and back again in 3 hours 55 minutes. How fast did she walk uphill?

- a) 2mph b) 3mph c) 4mph d) 5mph e) NOTA

4. Which of the following systems of equations has a solution that is consistent and independent.

a) $\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ b) $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
 d) $\begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ e) NOTA

5. Find the sum of all real values for x if: $\begin{vmatrix} x & -1 & 4 \\ 2 & x & -5 \\ -3 & 1 & x \end{vmatrix} = -53$.

- a) $-2 + \sqrt{13}$ b) 3 c) 2 d) -2 e) NOTA

6. Find the value of the determinant given: $\begin{vmatrix} 2i & -i & 3 \\ i & 4 & -2 \\ -3 & i & 4 \end{vmatrix}$ where $i = \sqrt{-1}$.

- a) $(25 + 26i)$ b) $(35 + 28i)$ c) $(15 + 32i)$ d) $(35 - 26i)$ e) NOTA

7. The area with vertices at $(-1, 3)$, $(4, 5)$, and (x, y) is 32.5. Which of the following is a possible point for (x, y) .

- a) $(9, -6)$ b) $(4, 14)$ c) $(-1, -4)$ d) $(-6, 15)$ e) NOTA

8. Find the intersection of the planes: $2x - 3y + z = 4$, $x - 3y + 5z = -10$, and $z = -3$

- a) $(1, 1, -3)$ b) $(2, -1, -3)$ c) $(\frac{4}{3}, -\frac{11}{9}, -3)$
 d) $(-1, -\frac{11}{3}, -3)$ e) NOTA

9. Find the value of xy in the following: $\begin{bmatrix} 3x & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x & y \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 42 & 4 \end{bmatrix}$.

- a) 4 b) 5 c) 6 d) 8 e) NOTA

10. For what value(s) of a would $AB = BA$, if $\begin{bmatrix} -2 & -4 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} a & 4 \\ a-9 & -2 \end{bmatrix}$

- a) 2 b) 3 c) 4 d) 5 e) NOTA

11. Which of the following products are defined for matrices $A_{2 \times 2}, B_{2 \times 3}, C_{3 \times 2}, D_{5 \times 2}$?
 I AB; II AC; III AD; IV BC.

- a) I only b) I, II only c) I, IV only d) I, II, IV only e) NOTA

12. Find: $\sum_{n=0}^3 \left(\begin{vmatrix} \sin \frac{\pi}{3} & 1 \\ 1 & \sin \frac{\pi}{3} \end{vmatrix}^n - \begin{vmatrix} 1 & \cos \frac{\pi}{3} \\ \cos \frac{\pi}{3} & 1 \end{vmatrix}^n \right)$

- a) $\frac{-31}{16}$ b) $\frac{113}{32}$ c) $\frac{-75}{32}$ d) $\frac{25}{8}$ e) NOTA

13. In the augmented matrix: $\begin{bmatrix} 3 & -2 & 4 & 15 \\ 2 & 1 & -3 & -1 \\ 1 & -3 & 1 & -8 \end{bmatrix}$ when solving using the reduced row

echelon form the second row would be reduced to which of the following:

- a) 0 1 0 3 b) 0 0 1 2 c) 0 1 0 5
 d) 1 0 0 -2 e) NOTA

14. The $\det \begin{bmatrix} x & 3 & 1 \\ 4 & 5 & 7 \\ 1 & -2 & x \end{bmatrix} = 32$, find the value of the positive x .

- a) 2 b) 2.5 c) $\frac{18}{7}$ d) $\frac{25}{7}$ e) NOTA

15. Which of the following is perpendicular to $3i - 4j + k$ is:

- a) $-3i + 4j - k$ b) $2i + 3j - 10k$ c) $-2i - 3j - 6k$
d) $3i + 2j + k$ e) NOTA

16. Which of the following operations or properties of matrices is false:

- a) $A + B = B + A$ b) $AA^{-1} = I$ (A is non-singular)
c) $A(B + C) = AB + BC$ d) $AB = BA$ e) NOTA

17. Find the cross product of the vectors $(2, 3, 5)$ and $(-5, 3, 1)$.

- a) $(-12, -27, 21)$ b) $(-10, 9, 5)$ c) $(-3, 6, 6)$
d) $(4, 1, -5)$ e) NOTA

18. Which of the following matrices is singular?

- a) $\begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 3 \\ 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$ d) $\begin{bmatrix} -1 & -3 \\ 1 & -3 \end{bmatrix}$ e) NOTA

19. Given the determinant $\begin{vmatrix} -1 & 1 & 2 & 2 \\ 2 & -2 & -4 & -3 \\ 3 & 4 & 4 & 5 \\ 3 & 3 & -6 & 1 \end{vmatrix}$, and using algebraic methods which of the

following would be the most direct method to find a row or column of three zeros.

- a) $-3R_3 + R_4$ b) $2C_1 + C_4$ c) $2C_1 + C_3$ d) $2R_1 + R_2$ e) NOTA

20. Which of the following is the dot product of the vectors $(-1, -3, 5)$ and $(3, -2, 1)$?

- a) 14 b) 10 c) 8 d) -4 e) NOTA

21. Let A be a matrix of order n, matrix B is obtained by an operation on A, and k be any real number, then which of the following statements is false.

- a) If 2 rows are interchanged then $\det(B) = -\det(A)$
b) If one column of A is multiplied by k, then $\det(B) = k\det(A)$
c) If one row of A is multiplied by k and added to another row then $\det(B) = \det(A)$
d) If one row or one column has all zeros then the $\det(A) = 0$
e) NOTA

22. Find the w value in the following system of equations: $3x - 2y + 4z = 4$

$$\begin{aligned} 2x + 4z - 3w &= 5 \\ -3y - 3z + 2w &= -3 \\ x + y - w &= 10 \end{aligned}$$

- a) $-\frac{17}{3}$ b) $-\frac{333}{11}$ c) $-\frac{9}{19}$ d) $-\frac{4}{3}$ e) NOTA

23. Find the measure of an angle between $2i - 5j + 2k$ and $-4i - j + 5k$ to the nearest tenth of a degree.

- a) 78.9° b) 79.2° c) 90.5° d) 91.2° e) NOTA

24. Given: $A = \begin{bmatrix} 3 & 1 \\ 2 & 7 \end{bmatrix}; B = \begin{bmatrix} 5 & 19 \\ 16 & 57 \end{bmatrix}$, with $AX = B$. Find the sum of the entries in X .

- a) -2 b) 6 c) 11 d) 14 e) NOTA

25. An airplane is flying on a compass heading of 340° at 325 mph. A wind is blowing with the bearing of 320° at 40 mph. What is the actual speed of the plane?

- a) 315 b) 335 c) 347 d) 363 e) NOTA

26. Which of the following is the magnitude of the vector $3i - 2j + 4k$?

- a) $\sqrt{5}$ b) 3 c) $\sqrt{21}$ d) $\sqrt{29}$ e) NOTA

27. A force of 450 lb motorcycle is on a hill inclined at 11° . What force is required to keep the motorcycle from rolling back down the hill? (Assume the force is parallel to the ground)

- a) 79.3 b) 85.9 c) 87.5 d) 89.2 e) NOTA

28. If $U = -3i - 2j + 5k$ and $W = 5i - 7j - 10k$ find V if $V = 2U - 3W$.

- a) $-21i + 17j + 40k$ b) $-90i - 25j - 20k$ c) $-21i + 17j - 20k$
d) $2i - 9j - 5k$ e) NOTA

29. Find the sum of the entries of the product of AB if $A = \begin{bmatrix} -2 & 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 5 & 3 \\ 9 & 8 \end{bmatrix}$.

- a) 30 b) 76 c) 111 d) 156 e) NOTA

30. Find the unit vector for $2i - 3j + 6k$.

a) $i - j + k$

b) $\frac{2}{\sqrt{31}}i - \frac{3}{\sqrt{31}}j + \frac{6}{\sqrt{31}}k$

c) $\frac{2}{7}i - \frac{3}{7}j + \frac{6}{7}k$

d) $\frac{1}{3}i - \frac{1}{2}j + k$

e) NOTA

Tie Breaker #1:

A characteristic equation of a square matrix \mathbf{A} is the equation $|\mathbf{A} - \lambda\mathbf{I}| = 0$. Find the characteristic equation

for $\mathbf{A} = \begin{bmatrix} 10 & 2 \\ 4 & 1 \end{bmatrix}$

Tie Breaker #2:

Find the plane that contains the points $(1, 0, -1)$, $(-5, 3, 2)$, and $(2, -1, 4)$