I

Lehigh 96 Calculus Individual Solutions

1) $(x-1)^2+(y+1)^2=4 \quad C_1(1,-1)$

$(x+3)^2+(y-2)^2=20 \quad C_2(-3,2)$

$d=\sqrt{(1+3)^2+(-1-2)^2}=5$ Ⓑ

2) $y'=-2x+5$; $0=-x^2+5x-6$

$(x-2)(x-3)=0$

$x=3,2$

$y'_{x=2}=1$

$y'_{x=3}=-1$ Ⓐ

3) $\lim\limits_{x\to0}\frac{\sin 2x}{\tan 3x}=\lim\limits_{x\to0}\frac{2\cos 2x}{3\sec^2 3x}=\frac{2}{3}$ Ⓔ

4) $\int x^2\cos(x^3+4)dx=\frac{1}{3}\sin(x^3+4)+C$

when $u=x^3+4$ and $\frac{du}{3}=x^2dx$ Ⓒ

5) $2yy'-3(xy'+y)+4x=0$

$4y'-3(3y'+2)+4(3)=0$

$-5y'+6=0$ Ⓐ

$y'=6/5$

6) $V=\frac{4}{3}\pi r^3\Rightarrow\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$

$A=4\pi r^2\Rightarrow\frac{dA}{dt}=8\pi r\frac{dr}{dt}$

$\left.\begin{array}{l}\frac{dV}{dt}=10\text{ in}^3/\text{sec}\\ r=4\end{array}\right\}$; $\frac{dV}{dt}=10=4\pi(4)^2\frac{dr}{dt}$

$\frac{dr}{dt}=\frac{5}{32\pi}$

$\therefore\frac{dA}{dt}=8\pi(4)\left(\frac{5}{32\pi}\right)=5\text{ in}^2/\text{sec}$ Ⓐ

7) $g(x)=\csc x\Rightarrow f'(x)=-\csc x\cot x$ Ⓒ

$g''(x)=\csc^3x+\csc x\cot^2x$

$g''(\pi/4)=(\csc\frac{\pi}{4})^3+\csc\frac{\pi}{4}(\cot\frac{\pi}{4})^2=3\sqrt{2}$

8) $f(x)=\ln x$ on $[1,2]$

$y_{ave}=\frac{1}{2-1}\int_1^2\ln x\,dx=x\ln x-x\Big|_1^2$

$=\ln 4-1$ Ⓑ

9) $H(x)=1996^{\cos^2x}$

$H'(x)=1996^{\cos^2x}(\ln 1996)(2\cos x)(-\sin x)$

$H'(\pi/4)=1996^{(\cos\pi/4)^2}(\ln 1996)(-\sin 2(\pi/4))$

$H'(\pi/4)=-\sqrt{1996}\,\ln 1996$ Ⓓ

10) Area (ellipse) = πab

$a=2,\ b=1\Rightarrow A=2\pi$

Half area would be π. Ⓑ

11) $L=\int_0^{\pi/4}\sqrt{1+(-\tan x)^2}\,dx$ where

$y'=-\tan x$

$L=\int_0^{\pi/4}\sec x\,dx=\ln|\sec x+\tan x|\Big|_0^{\pi/4}$

$L=\ln(\sqrt{2}+1)$ Ⓐ

12) $y=x\sin x$

$y'=x\cos x+\sin x$

$y'''=-x\cos x-\sin x-2\sin x$

$y'''=-x\cos x-3\sin x$ Ⓒ

13) l'Hopital's Rule (twice)

$\lim\limits_{x\to0}\frac{2e^{2x}+\sin x-3(x+1)^{-1}+\cos x}{3x^2-2-2xe^{x^2}+2}$

$=\lim\limits_{x\to0}\frac{4e^{2x}+\cos x+3(x+1)^{-2}-\sin x}{6x-4x^2e^{x^2}-2e^{x^2}}$

$=-4$ Ⓓ

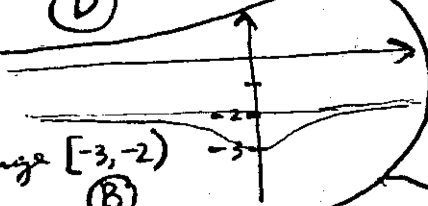
14) $h''(x)=\frac{(x^2+2)^2(-2)-(-2x)(2)(x^2+2)(2x)}{(x^2+2)^4}$

$h''(x)=\frac{-2x^2-4+8x^2}{(x^2+2)^3}=0\Rightarrow x=\pm\sqrt{\frac{2}{3}}$

$h''(x)$ ++++ $-\sqrt{2/3}$ ---- 0 ---- $\sqrt{2/3}$ +++++

$(-\infty,-\sqrt{\frac{2}{3}})\cup(\sqrt{\frac{2}{3}},\infty)$ Ⓒ

15)



Range $[-3,-2)$ Ⓑ

16) $f'(x)=4x^3-3x^2$

$4x^3-3x^2=\frac{f(1)-f(0)}{1-0}=0$

$x=0,\ x=3/4$ Ⓐ

17) $xe^{xy}=2$

$xe^{xy}\left[x\cdot1+y\frac{dx}{dy}\right]+e^{xy}\cdot\frac{dx}{dy}=0$

$(xye^{xy}+e^{xy})\frac{dx}{dy}=-x^2e^{xy}$ Ⓑ

$\frac{dx}{dy}=\frac{-x^2\cancel{e^{xy}}}{e^{xy}(xy+1)}=\frac{-x^2}{(xy+1)}$

18) $\lim\limits_{x\to\infty}(\sqrt{x^2+ax}-\sqrt{x^2+bx})$

multiply by conjugate

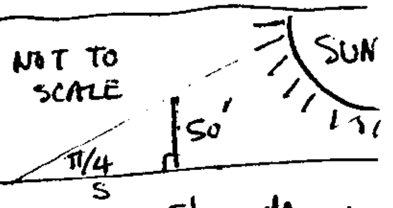
$=\lim\limits_{x\to\infty}\frac{x^2+ax-x^2-bx}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}=$ Ⓐ

$=\lim\limits_{x\to\infty}\frac{a-b}{\sqrt{1+a/x}+\sqrt{1+b/x}}=\frac{a-b}{2}$

19) $\int\frac{e^x\,dx}{2+e^x}=\ln(2+e^x)+C$ Ⓓ

where $u=2+e^x$ and $du=e^xdx$

20)



$\tan\theta=\frac{50'}{s}=50s^{-1}$; $\frac{d\theta}{dt}=-\frac{1}{4}$

$\sec^2\theta\frac{d\theta}{dt}=-50s^{-2}\frac{ds}{dt}$

$\frac{ds}{dt}=-\frac{s^2}{50}\cdot\frac{1}{\cos^2\theta}\frac{d\theta}{dt}$

$\frac{ds}{dt}_{\theta=\pi/4}=-\frac{50^2}{50}\cdot\frac{1}{(\frac{\sqrt{2}}{2})^2}\cdot(-1/4)$

$\frac{ds}{dt}=25$ ft/hr. Ⓑ

21) $(f\cdot g)'=f\cdot g'+g\cdot f'$

$(f\cdot g)'(3)=(f(3))(g'(3))+(g(3))(f'(3))$

$=(2)(-4)+(3)(-1)$ Ⓐ

$=-8-3=-11$

22) $C(x)=\frac{1}{4}x^2+35x+25$

$p(x)=50-\frac{1}{2}x$

$P=p\cdot x-C(x)=15x-\frac{3}{4}x^2-25$

$P'(x)=15-\frac{3}{2}x\Rightarrow x=10$; $P'(x)$ ++++ 10 ----

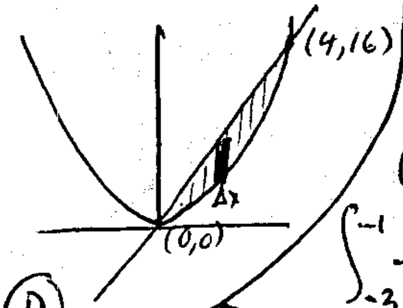
Max profit when $x=10$ Ⓐ

Calculus Individual Solutions

23) $y = x^2 = 4x$

$x^2 - 4x = 0$

$(x-4) = 0$



$A = \int_0^4 4x - x^2 dx$

$A = \left(2x^2 - \frac{x^3}{3}\right)\Big|_0^4 = 32/3$ Ⓓ

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24) $g''(x) = \frac{[(1+x^2)(-2x) - (1-x^2)(4x)]}{(1+x^2)^3}$ Ⓒ

$g''(x) = \frac{-6x + 2x^3}{(1+x^2)^5} = 0 \Rightarrow x = 0, \sqrt{3}, -\sqrt{3}$

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25) $\int_3^b \sqrt{2x+3}\, dx = \frac{98}{3}$

When $u = 2x+3$ and $\frac{du}{2} = dx$,

we have $\left(\frac{2x+3}{3}\right)^{3/2}\Big|_3^b = \frac{98}{3}$

$(2b+3)^{3/2} - 27 = 98$

$(2b+3)^{3/2} = 125$ Ⓑ

$b = 11$

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26) $g(x) = x^2e^x$

$g'(x) = xe^x(x+2)$

$g''(x) = e^x(x^2+4x+2)$

$g'(x)$ ++++ , - - - , + + + + + + + + + + →
-2 0

$g''(x)$ ++ - - - - , + + + + + + + + →
$-2-\sqrt{2}$ $-2+\sqrt{2}$

Ⓓ

increasing: $(-\infty, -2] \cup [0, \infty)$

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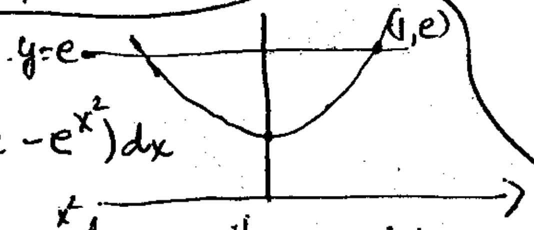
27)



$V = 2\pi \int_0^1 x(e - e^{x^2}) dx$

$= 2\pi \int_0^1 xe - xe^{x^2} dx$ with $u = x^2$ & $du = 2x dx$

$= 2\pi \left[\frac{ex^2}{2}\right]\Big|_0^1 - \frac{2\pi}{2}\int e^u du$ Ⓐ

$= \pi e - \pi e + \pi = \pi$

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28) $\int_{-3}^{2} |x^3 - x| dx$ simplifies on graphing calculator to 75/4

Ⓓ

Algebraically:

$\int_{-3}^{-1} -(x^3-x)dx + \int_{-1}^{0} (x^3-x)dx + \int_0^1 -(x^3-x)dx + \int_1^2 (x^3-x)dx =$

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29) $g(x) = x^3 - 2x^2 + x + 3$

$g'(x) = 3x^2 - 4x + 1 = 0 \Rightarrow x = 1/3, 1$

$g'(x)$ + + + + + + , - - - - - , + + + + →
1/3 1

max when $x = 1/3$ & min when $x = 1$

$g(1/3) = 85/27$ $g(1) = 3$ Ⓐ

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30) $\int \frac{x^4 + 8x^2 + 8\, dx}{x^3 - 4x} = \int x dx + \int \frac{12x^2+8}{x^3-4x} dy$

$\frac{12x^2+8}{x^3-4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$

Using partial fractions yields

$A = -2$

$B = 7$

$C = 7$

$\therefore \int x dx + \int \frac{-2dx}{x} + \int \frac{7dx}{x-2} + \int \frac{7dy}{x+2}$ Ⓒ

translates to $\frac{x^2}{2} - 2\ln|x| + 7\ln|x-2| + 7\ln|x+2| + C$