Calculus Individual Test
February 20, 1993

NOTA means None of the Above

1: Evaluate $\lim_{x\to 2}\frac{x^2 + x - 6}{x^2 - 4}$.

A. 1 B. $\frac{3}{2}$ C. $\frac{5}{4}$ D. 0 E. NOTA

2: Find the domain of $f(x) = \sqrt{5 - 4x - x^2}$.

A. $[-5,1]$ B. $[-5,\infty]$ C. $[1,\infty]$ D. $[-\infty,-5] \cup [1,\infty]$

E. NOTA

3: Find the function f(x) if $f'' = x + 2$, $f'(0) = 3$, and $f(0) = -1$.

A. $\frac{1}{6}x^3 + x^2 + 3x - 1$ B. $\frac{x^3}{6} + 2x^2 + C$

C. $x^3 + 6x^2 + 18x - 6$ D. $\frac{1}{6}x^3 + x^2 + \frac{21}{2}x + \frac{61}{6} + C$

E. NOTA

4: Find $f'(x)$ if $f(x) = \sqrt{4 - \sqrt{4 + x}}$.

A. $\frac{1}{4(\sqrt{4 - \sqrt{4 + x}})\sqrt{4 + x}}$ B. $\frac{1}{2\sqrt{4 - \sqrt{4 + x}}}$

C. $\frac{1}{2\sqrt{4 - \sqrt{(4 + x)^3}}}$ D. $\frac{-1}{\sqrt{4 - \sqrt{4 + x}}\sqrt{4 + x}}$

E. NOTA

5: Find the absolute maximum of $f(x) = \frac{(\ln x)^2}{x}$ on $[1,e]$.

A. 0 B. 1 C. $\frac{1}{2}$ D. e E. NOTA

6: The radius of a sphere is measured to be 3.0 inches. If the measurement is correct to within 0.01 inch, use differentials to estimate the propagated error in the volume of the sphere.

A. ± 0.000001 in^3 B. $\pm 0.36\pi$ in^3

C. $\pm 0.036\pi$ in^3 D. ± 0.06 in^3

E. NOTA

7: A machine is rolling a metal cylinder under pressure. The radius of the cylinder is decreasing at a constant rate of 0.05 inches per second and the volume V is 128π cubic inches. At what rate is the length h changing when the radius r is 2.5 inches?

A. 20.48 in / sec. B. -0.8192 in / sec.

C. -16.38 in / sec. D. 0.8192 in / sec. E. NOTA

8: Find $\int \frac{x^4+3x^3-2x-3}{x^2+3x}\,dx$.

A. $\frac{1}{5}x^5+\frac{3}{4}x^4-x^2-3x+\ln|x^2+3x|+C$ B. $\frac{1}{3}x^3-\ln|x^2+3x|+C$

C. $\frac{1}{3}x^3+\frac{3}{2}x^2+C$ D. $\frac{1}{3}x^3+\frac{3}{2}x^2-\frac{11}{3}x+C$

E. NOTA

9: For what values of k is the following a continuous function?

$$f(x)=\begin{cases}\dfrac{\sqrt{7x+2}-\sqrt{6x+4}}{x-2} & \text{if } x\geq -\frac{2}{7} \text{ and } x\neq 2\\ k & \text{if } x=2\end{cases}$$

A. $-\frac{2}{7}$ B. $-\frac{16}{7}$ C. 2 D. $\frac{1}{8}$ E. NOTA

10: Evaluate $\int \frac{dx}{2+9x^2}$.

A. $\frac{1}{18}\ln|2+9x^2|+C$ B. $\frac{1}{3\sqrt{2}}\arctan\frac{3x}{\sqrt{2}}+C$

C. $2x+3x^3+C$ D. $\arcsin\frac{3x}{\sqrt{2}}+C$

E. NOTA

11: The range of the function $f(x) = \frac{x^2 - 4}{x + 2}$ is

A. $(-\infty,\infty)$ B. $(-\infty,-2)\cup(-2,\infty)$ C. $(-\infty,4)\cup(4,\infty)$

D. $(-\infty,-4)\cup(-4,\infty)$ E. NOTA

12: Find the absolute maximum value of f in the interval $[-1,2]$ if $f(x) = -2x^3 + 3x^2$.

A. -4 B. 5 C. 0 D. 1 E. NOTA

13: Find $\frac{dy}{dx}$ if $\tan(x+y) = x$.

A. $\frac{1 - \sec^2 x}{\sec^2 y}$ B. $-\sin^2(x+y)$ C. $-\tan^2(x+y)$

D. $\sec^2(x+y)$ E. NOTA

14: If $\int_0^k (2kx - x^2)\,dx = 144$, then k =

A. 12 B. 6 C. -6 D. -12 E. NOTA

15: A deposit of $1,000.00 is made into a fund with an annual interest rate of 10 percent. Find the time (in years) necessary for the investment to double if the interest is compounded continuously.(Round to two decimal places)

A. 10 years B. 7.23 years C. 6.93 years

D. 20 years E. NOTA

16: If $f(x) = 2x^2 + 4$, which of the following will calculate the derivative of f(x)?

A. $\frac{[2(x+\Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$ B. $\lim_{\Delta x \to 0} \frac{[2(x+\Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

C. $\lim_{\Delta x \to 0} \frac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$ D. $\frac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

17: Evaluate $\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x^2-2x}$.

A. $\frac{1}{3}$ B. 0 C. 1 D. ∞ E. NOTA

18: Evaluate the indefinite integral: $\int (\ln x)^4 dx$

A. $x(\ln x)^4 - 4\int (\ln x)^3 dx$ B. $x(\ln x)^4 - 4(\ln x)^3 + C$

C. $\frac{4}{x(\ln x)^4} + C$ D. $\frac{1}{4}(\ln x)^5 + C$ E. NOTA

19: Find an equation for the tangent line to the graph of $f(x) = -2x^2 + 2x + 3$ at the point where $x = 1$.

A. $y = -4x + 2$ B. $2x + y - 1 = 0$

C. $y = -4x^2 + 2x + 1$ D. $2x + y = 5$ E. NOTA

20: Find $\frac{dy}{dx}$ if $f(x) = \frac{\sqrt{x+2}}{\sqrt{x-1}}$.

A. $\frac{\sqrt{x+2}}{2\sqrt{x-1}}$ B. $\frac{-3}{2\sqrt{x+2}(\sqrt{x-1})^3}$

C. $\frac{1}{2\sqrt{x+2}(\sqrt{x-1})}$ D. $\frac{\sqrt{x+2}}{2(x-1)}$ E. NOTA

21: Evaluate $\int_0^1 \frac{x}{3+5x} dx$.

A. $\ln\frac{8}{3}$ B. $5\ln\frac{8}{3}$ C. $\frac{1}{5} - \frac{3}{25}\ln\frac{8}{3}$ D. $\frac{1}{8}$ E. NOTA

22: Find the area of the region bounded by $y = x - 1$ and $y = 2x^2 - 5x + 3$.

A. 3 B. $\frac{1}{3}$ C. $\frac{1}{6}$ D. $\frac{3}{2}$ E. NOTA

23: Find $\frac{d^2y}{dx^2}$ if $y = \sin^2 x$.

A. $2 \sin x \cos x$ B. $2 \cos x$ C. $2 \sin x$

D. $2 \cos 2x$ E. NOTA

24: $\lim_{x \to 6} f(x)$ is non-existent if $f(x) =$

A. $\frac{x^2 - 36}{x - 6}$ B. $\sin (x - 6)$ C. $\frac{x}{x^2 - 6x}$

D. $\frac{x - 6}{x}$ E. NOTA

25: Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x^3$ and the line $y = x$, between $x = 0$ and $x = 1$ about the y-axis.

A. $\pi \int_0^1 (x^2 - x^4)\, dx$ B. $\pi \int_0^1 (y^{\frac{1}{3}} - y)^2\, dy$

C. $\pi \int_0^1 (x^4 - x^2)\, dx$ D. $\pi \int_0^1 (y^{\frac{2}{3}} - y^2)\, dy$

E. NOTA

26: Find all points of inflection of the function $f(x) = x^4 + x^3$.

A. (0,0) and $\left(-\frac{1}{2}, \frac{1}{16}\right)$ B. $\left(-\frac{1}{2}, -\frac{1}{16}\right)$

C. (0,0) D. (0,0) and $\left(-\frac{3}{4}, -\frac{27}{256}\right)$

E. NOTA

27: Which of the following functions has a horizontal asymptote at $y = 2$?

A. $\frac{2x^2 - 6x + 1}{1 + x^2}$ B. $\frac{2x}{\sqrt{x - 2}}$

C. $\frac{x - 2}{3x - 5}$ D. $\frac{2x - 1}{x^2 + 1}$

E. NOTA

28: Which statement is NOT true of the graph of $f(x) = (x+2)(x-1)^2$?

A. f has a relative minimum at (1,0).

B. f has an intercept at (1,0).

C. f has a relative maximum at (-1,4).

D. f has a point of inflection at (1,0).

E. NOTA

29: Find all intervals on which the function is concave upward: $f(x) = \frac{6}{x^2+3}$.

A. $(-\infty,\infty)$ B. $(-\infty,-1)$ and $(1,\infty)$ C. $(-\infty,0)$ and $(0,\infty)$

D. $(1,\infty)$ E. NOTA

30: Which of the following statements is NOT true of $f(x) = \sqrt{x^2-49}$?

A. f is continuous on the interval $(-\infty, 7]$.

B. f is continuous at $x = 14$.

C. f is continuous on the interval $[-7,7]$.

D. f is continuous on the interval $[7,\infty)$.

E. NOTA