

ALGEBRA IITEAM ANSWERSREGIONALFEBRUARY 1997

1. $\frac{3}{2}, 4$

2. 0

3.
$$\frac{2\sqrt{3} - 3\sqrt{2} + \sqrt{30}}{3}$$

4. $-2, \frac{1}{3}, 1$

5. 29

6. $\frac{21}{4}$ or $5\frac{1}{4}$ or 5.25

7. 25

8. 21

9. -28

10. 3138

11. 728

12. $-\frac{1}{2}$

13. $\frac{7}{8}$

14. -21

15.
$$\frac{17\sqrt{5}}{5}$$

$$1) 2^{2(x^2-6)} = 2^{-5x}, \quad 2^{2x^2-12} = 2^{-5x}, \quad 2x^2+5x-12=0, \quad (2x-3)(x+4)=0 \quad x = \frac{3}{2} \text{ or } -4 \Rightarrow \left(\frac{3}{2}, -4\right)$$

$$2) x = \frac{\begin{vmatrix} -7 & 2 & -1 \\ -3 & 3 & 2 \\ -9 & 2 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 2 \\ 1 & 2 & -2 \end{vmatrix}} = \frac{1}{1} = 1, \quad y = \frac{\begin{vmatrix} 1 & -7 & -1 \\ 2 & -3 & 2 \\ 1 & -9 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 2 \\ 1 & 2 & -2 \end{vmatrix}} = \frac{-3}{1} = -3, \quad z = \frac{\begin{vmatrix} 1 & 2 & -7 \\ 2 & 3 & -3 \\ 1 & 2 & -9 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & 2 \\ 1 & 2 & -2 \end{vmatrix}} = \frac{2}{1} = 2 \quad x=1, y=-3, z=2$$

$$1 + (-3) + 2 = 0 \quad \boxed{\text{ans: } 0}$$

$$3) \frac{4}{\sqrt{3}+\sqrt{5}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{5}+\sqrt{2}}{\sqrt{3}+\sqrt{5}+\sqrt{2}} \Rightarrow \frac{4(\sqrt{3}+\sqrt{5}+\sqrt{2})}{2(3+\sqrt{15})} \cdot \frac{3-\sqrt{15}}{3-\sqrt{15}} \Rightarrow \frac{6\sqrt{3}+6\sqrt{5}+6\sqrt{2}-2\sqrt{45}-2\sqrt{75}-2\sqrt{30}}{-6} \Rightarrow \boxed{\text{ans: } \frac{2\sqrt{3}-3\sqrt{2}+\sqrt{30}}{3}}$$

$$4) \text{ possible rational roots } \pm 1, 2, 4, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3} \quad f(1) = 6+4-14+4 = 0 \quad x-1 \quad \begin{array}{r} 6 \quad 4 \quad -14 \quad 4 \\ -6 \quad 10 \quad -4 \\ \hline 6 \quad 10 \quad -4 \quad 0 \end{array} \quad 6x^2+10x-4$$

$$2(3x-1)(x+2) \quad x = \frac{1}{3}, -2$$

$$\boxed{\text{ans: } (-2, 1, \frac{1}{3})}$$

$$5) A = x = \sqrt{6+3\sqrt{2}} \Rightarrow x^2 = 6 + \sqrt{6\sqrt{2}} \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x+2)(x-3) \quad x \neq -2, \quad \boxed{x=3}$$

$$B = 6 + 3 + \frac{3}{2} + \frac{3}{2} + \dots = \frac{6}{1-\frac{1}{2}} = \boxed{12} \quad C = 6^{2 \log_6 2} = 6^{\log_6 4} = \boxed{4}$$

$$D = 5C_2 x^{5-2} y^2 \Rightarrow \frac{5!}{2!3!} = \boxed{10} \quad 3 + 12 + 4 + 10 = 29 \quad \boxed{\text{ANS: } 29}$$

$$6) \frac{\log 3^6}{\log 5^3} \cdot \frac{\log 7^3}{\log 2^3} \cdot \frac{\log 5^4}{\log 6^4} \cdot \frac{\log 2^7}{\log 7^2} \cdot \frac{\log 6^3}{\log 3^4} = \frac{6 \log 3}{3 \log 5} \cdot \frac{3 \log 7}{3 \log 2} \cdot \frac{7 \log 5}{2 \log 6} \cdot \frac{3 \log 6}{4 \log 3} = \frac{6 \cdot 3 \cdot 4 \cdot 7 \cdot 3}{3 \cdot 3 \cdot 4 \cdot 2 \cdot 4} = \frac{42}{8} = \frac{21}{4} \text{ or } 5.25$$

$$7) |(2+i)^4| \Rightarrow |(2-i)^4| \Rightarrow (2-i)^4 = -7 - 24i \Rightarrow |-7-24i| = \sqrt{(-7)^2 + (-24)^2} = \sqrt{625} = \boxed{25 \text{ ans}}$$

$$8) (x-0)^2 = -4a(y-25) \Rightarrow 20^2 = -4a(0-25) \text{ so } a=4 \quad (x-0)^2 = -16(y-25) \Rightarrow 8^2 = -16(y-25) \text{ so } y=21$$

$$9) -\frac{b}{a} = \frac{16}{3} \text{ so } b = -16 \quad \frac{c}{a} = -4 \Rightarrow \frac{c}{3} = -4 \quad c = -12 \quad -16 + (-12) = -28 \quad \boxed{\text{ans: } -28} \quad \boxed{\text{ans: } 21+4}$$

$$10) A = 2^3 \cdot 3^5 \cdot 5^1 \Rightarrow (3+1)(5+1)(1+1) = 4 \cdot 6 \cdot 2 = \underline{48} \quad B = 1+3+6+10+15 = \underline{35} \quad C = 1^3+2^3+\dots+10^3 = 3025$$

$$D = 6P_2 = \frac{6!}{4!} = 30 \quad 48 + 35 + 3025 + 30 = \boxed{3138 \text{ ans:}}$$

$$11) A = \frac{8!}{2!3!} = \underline{3360} \quad B. \text{ FACE CARD (K, Q, J)} \quad \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}$$

$$C. \frac{(5-1)!}{2} = 12 \quad D. 5 \text{ to } 3 \text{ lose } \frac{5}{8} \quad \frac{3360}{12 \cdot 8/13 \cdot 5/8} = \boxed{728}$$

$$12) A+B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} -5/2 & 3/2 \\ 2 & -1 \end{vmatrix}} \begin{bmatrix} 5/2 & -3 \\ 2 & -1 \end{bmatrix} \quad \frac{5}{2} - 3 = -\frac{1}{2} \quad \boxed{\text{ans: } -\frac{1}{2}}$$

$$13) A = \text{eccentricity of parabola} = \frac{1}{2} \quad B = \frac{(x+2)^2}{2} + \frac{(y+1)^2}{1} = 1 \quad \frac{c}{a} = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$C = \frac{x^2}{16} - \frac{y^2}{12} = 1 \quad c = \sqrt{16+12} = 2\sqrt{7} \quad \frac{2\sqrt{7}}{4} = \frac{\sqrt{7}}{2} \quad (ABC)^2 \left(1 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{7}}{2}\right)^2 = \frac{14}{16} = \frac{7}{8} \text{ ans}$$

$$14) \text{ Sum } -\frac{(-3)}{1} = 3 \quad \text{product } -\frac{(-24)}{1} = -24 \quad 3 + (-24) = \boxed{-21 \text{ ans:}}$$

$$15) \frac{|1(-3) + -2(4) - 6|}{\sqrt{(1)^2 + (-2)^2}} = \frac{|-17|}{\sqrt{5}} = \frac{17}{\sqrt{5} \sqrt{5}} = \boxed{\frac{17\sqrt{5}}{5} \text{ ans:}}$$