FAMAT Regional Competition **February 2001**
Algebra 2-THETA Individual Test
NOTA denotes None Of These Answers

1. If x is a real number, the values of $x^2 - 2x$ can never be less what value?

   A) 1 B) 0 C) -1 D) -2 E) NOTA

2. For $xy \neq 0$, simplify $\dfrac{x+y}{x^{-1}+y^{-1}}$

   A) $x^2y + xy^2$ B) $(x+y)^2$ C) $x^2 + y^2$ D) xy E) NOTA

3. Find the sum of the coefficients in the expansion of $(2x - 3y)^{10}$

   A) 1 B) 2^7 C) 2^8 D) $2^7 - 3^7$ E) NOTA

4. How many lines can be drawn between 30 points no three which lie on the same line?

   A) 15 B) 30 C) 435 D) 3^{28} E) NOTA

5. Given the graphs of $2x + 3y = 7$ and $y = \dfrac{8-2x}{3}$ which of the following are true?

   i. The lines are parallel
   ii. The lines are the same
   iii. The lines are perpendicular
   iv. The lines have the same y-intercept

   A) only i B) only ii C) only iii and iv D) only iii E) NOTA

6. Find the domain of $f(x) = \sqrt{x^2 + 2x - 3}$

   A) $\{x: x \in \text{Reals}\}$ B) $\{x: x \geq 0\}$ C) $\{x: -3 \leq x \leq 1\}$
   D) $\{x: x \leq -3 \text{ or } x \geq 1\}$ E) NOTA

7. $a^{\log_a b} + b^{\log_b a} = ?$ if $a > 0, b > 0$

A) $a + b$ B) ab C) $\frac{(\log b)^2 + (\log a)^2}{\log a \log b}$
D) $\log_a b$ E) NOTA

8. Simplify $\frac{5+2i}{6-i}$, where $i = \sqrt{-1}$

A) $\frac{32-7i}{37}$ B) $\frac{28+17i}{37}$ C) $\frac{32+17i}{35}$ D) $\frac{28+17i}{35}$ E) NOTA

9. If $(a-b)^2 = 25$ and $a^2 + b^2 = 73$ then $ab = ?$

A) 6 B) 12 C) 24 D) 48 E) NOTA

10. The graph of $4x^2 + y^2 - 16x - 2y + 17 = 0$ is

A) a parabola B) a point C) a hyperbola D) an ellipse E) NOTA

11. Find $g(f(x))$ if $f(x) = 2\log_3 x$ and $g(x) = 3^x$, for $x > 0$.

A) x^2 B) $2x$ C) 3^{x^2} D) 3^{22x} E) NOTA

12. What is the radius of the circle defined by the equation $4x^2 + 4y^2 - 8x + 4y - 27 = 0$?

A) 27 B) 8 C) $3\sqrt{3}$ D) $2\sqrt{2}$ E) NOTA

13. Describe the nature of the solutions of $x^2 - 2x + 2 = 0$

A) both real solutions
B) only one real solution
C) only one complex nonreal solution
D) both complex nonreal solutions
E) NOTA

14. Write $K^4 - W^3K - WK^3 + K^2W^2$ in completely factored form

A) $K(K^3-W^3-WK^2) + KW^2$
B) $K(K^2 + W^2)(K - W)$
C) $K^2[K(K-W) - K(K + W)]$
D) $K[K^2(K-W) - W^2(K+W)]$
E) NOTA

15. Reduce $\dfrac{a^8 - b^{16}}{b^{16} - a^8}$, $a \neq b^2$

A) -1 B) 1 C) $a^2 - b^4$ D) $b^2 - a^2$ E) NOTA

16. Which of the following numbers is not a factor of $(3^{12} - 2^{12})$?

A) 13 B) 19 C) 35 D) 63 E) NOTA

17. If $f(x) = 2^x - 5$ then $f^{-1}(x) = ?$ (for $x > -5$)

A) $2^x + 5$ B) $\dfrac{1}{2^x - 5}$ C) $\log_2(x + 5)$ D) $\log_2 \dfrac{x}{5}$ E) NOTA

18. When $x^3 - 8x^2 + 13x - 20$ is divided by $x - 3$, the remainder is

A) -15 B) -26 C) -20 D) -14 E) NOTA

19. The range of $f(x) = \dfrac{1}{\sqrt{x^2 - 4}}$ is

A) All positive reals B) all Reals C) reals such that $-2 < y < 2$

D) reals such that $y < -2$ or $y > 2$ E) NOTA

20. A freight train one mile long, traveling 5 mph goes through a tunnel, one mile long. How many minutes does it take to pass through the tunnel?

A) 1 B) 5 C) 12 D) 24 E) NOTA

21. If it takes six minutes to cut a board into three pieces, how long would it take to cut the same type board into four pieces?

A) 8 min B) 9 min C) 10 min D) 12 min E) NOTA

22. A crab is on one end of a floating log, which is eight feet long and has a circumference of three feet. The log starts rolling and makes 2 complete revolutions as the crab stays on top of the log and travels to the other end. How far did the crab travel?

A) 8' B) 10' C) 14' D) 18.84' E) NOTA

23. Solve $7^{x+1} = 13$ for x

A) $\log \frac{13}{7} - 1$ B) $\frac{\log 7}{\log 13} - 1$ C) $\frac{\log 13 - \log 7}{\log 7}$ D) $\frac{\log 7 - \log 13}{\log 13}$ E) NOTA

24. If $x^2 + y^2 = 36$ and $x^2 + y = 6$ are graphed on the same set of axes, how many points do they have in common?

A) 3 B) 2 C) 1 D) 0 E) NOTA

25. The only possible rational roots for $y = 3x^3 + 2x^2 - Ax + 2$, where A is an integer, are

A) $\frac{\pm 2}{3}, \frac{\pm 3}{2}$ B) $\frac{\pm 3}{2}$ C) $\frac{\pm 1}{3}, \frac{\pm 2}{3}, \pm 1, \pm 2$ D) $\pm 1, \pm 2$ E) NOTA

26. A rule for the nth term of $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25} \ldots$ is

A) $\frac{n}{n^2}$ B) $\frac{n+1}{n+2}$ C) $\frac{n}{2n}$ D) $\frac{n}{(n+1)^2}$ E) NOTA

27. What is the minimum point for the following equation $f(x) = 2x^2 + 4x + 5$?

A) (1,3) B) (0,5) C) (-1, 3) D) (2,0) E) NOTA

28. Your home is built on a square lot. To add more space to your yard, you increase one side of the property by purchasing a 4 foot strip of land, adjacent to your property. The area of the new lot is 9600 square feet. What are the dimensions of the new lot?

A) 96 x 100 B) 94 x 98 C) 128 x 75 D) 60 x 160 E) NOTA

29. Solve the equation: $-5x^2 - 5x + 9 = 0$

A) $\frac{3}{2} \pm \frac{\sqrt{19}}{2}$ B) $\frac{-1}{2} \pm \frac{\sqrt{205}}{10}$ C) $2 \pm \frac{\sqrt{26}}{2}$ D) $1 \pm \sqrt{6}$ E) NOTA

30. Let $f(x) = x - 4$, and $g(x) = -x + 6$ find $g(f(0)) - f(g(2))$

A) 0 B) 1 C) 8 D) 10 E) NOTA