FAMAT REGIONAL COMPETITION, FEBRUARY --- ALGEBRA II ANSWERS
2001

1	C
2	D
3	A
4	C
5	A
6	D
7	A
8	B
9	C
10	B
11	A
12	D
13	D
14	B
15	A
16	D
17	C
18	B
19	A
20	D
21	B
22	B
23	C
24	A
25	C
26	D
27	C
28	A
29	B
30	D

Team Answers February 2001

1. 420
2. 16
3. 10.5 or 21/2
4. 14.75 or 59/4
5. 64
6. 99
7. 821.5 or 1643/2
8. 2
9. 100
10. 5
11. 250
12. 586
13. C
14. 3
15. -1.5 or –3/2

FAMAT REGIONAL COMPETITION, FEBRUARY --- ALGEBRA II SOLUTIONS

1. Let $y = x^2 - 2x$. Complete the square to get $y = (x-1)^2 - 1$ which is a parabola with a minimum point at (1, -1). Thus y is never less than –1, Thus C.

2. $\dfrac{x+y}{\left(\frac{y}{y}\right)\frac{1}{x}+\left(\frac{x}{x}\right)\frac{1}{y}} = \dfrac{x+y}{\frac{y+x}{xy}} = xy =$ Thus, D.

3. $(2x-3y)^{10} = [2(1)-3(1)]^{10} = [2-3]^{10} = (-1)^{10} = 1$, Thus A.

4. ${}_{30}C_2 = \dfrac{30!}{2!(30-2)!} = \dfrac{(30!)(29!)(28!)}{2(28!)} = (15)(29) = 435$ OR each point is connected to 29 others, (30)(29) but divide by 2 since you don't count lines twice, thus C.

5. 3y = -2x + 7, $y = \dfrac{-2}{3}x + \dfrac{7}{3}$ And 3y = 8-2x, $y = \dfrac{-2}{3}x + \dfrac{8}{3}$. Slopes are the same, therefore parallel but with different y-intercepts. Thus, A.

6. $y = \sqrt{x^2 + 2x - 3}$, $y = \sqrt{(x+3)(x-1)}$. So $\{x \le -3 \text{ or } x \ge 1\}$, Thus D.

7. $a^{\log_a b} + b^{\log_b a}$ (using log properties) = $b + a$, Thus, A.

8. $\dfrac{5+2i}{6-i} \bullet \dfrac{6+i}{6+i}$ (multiply by the conjugate, which results in $\dfrac{30+17i+2i^2}{36-i^2} = \dfrac{30-2+17i}{36+1} = \dfrac{28+17i}{37}$, Thus, B.

9. $a^2 - 2ab + b^2 = 25$, $a^2 + b^2 = 73$. Subtract the equations to get $-2ab = -48$, $ab = 24$, Thus C.

10. $4x^2 - 16x + \_\_ + y^2 - 2y + \_\_ = -17$ (completing the square), $4(x^2 - 4x + 4) + y^2 - 2y + 1 = -17 + 17$, $4(x-2)^2 + 1(y-1)^2 = 0$, is a point @ (2, 1)., Thus B.

11. $g(f(x)) = 3^{\log_3 x^2} = x^2$, Thus A.

12. $4x^2 - 8x + \_\_ + 4y^2 + 4y + \_\_ = 27$. $4(x^2 - 2x + 1) + 4(y^2 + y + 0.25) = 27 + 4 + 1$, $(x-1)^2 + (y + \frac{1}{2})^2 = 8$. Thus $r = 2\sqrt{2}$, Thus D.

13. Use the discriminant $(b^2 - 4ac)$, $(-2)^2 - (4)(1)(2) = 4-8 = -4$. Since discrim. is negative, 2 complex solutions, Thus D.

14. $K[K^3 - W^3 - K^2W + KW^2] = K[K^2(K-W) + W^2(K-W)] = K[(K^2 + W^2)(K-W)] = (K)(K-W)(K^2 + W^2)$, Thus B.

15. $\dfrac{a^8 - b^{16}}{b^{16} - a^8} = \dfrac{a^8 - b^{16}}{-(a^8 - b^{16})} = -1$, Thus A.

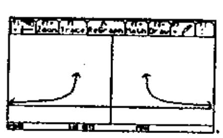
16. $3^{12} - 2^{12} = (3^6 - 2^6)(3^6 + 2^6)$. $(3^2 - 2^2)(3^4 + 3^2 2^2 + 2^4)(3^2 + 2^2)(3^4 - 3^2 2^2 + 2^4)$.
(9-4)(81+36+16)(9+4)(81-36+16) = (5)(133)(13)(61) = (5)(7)(19)(13)(61), Thus D.

17. First step is to switch {x} and {y} to yield $x = 2^y - 5$, then solve for "y". $x + 5 = 2^y$. Use logs to yield $\dfrac{\log(x+5)}{\log 2} = \dfrac{y \log 2}{\log 2}$ (Apply the reverse of the change of base formula to yield $y = \log_2(x+5)$, Thus C.

18. Use synthetic division:

3	1	-8	13	-20
		3	-15	-6
	1	-5	-2	-26

Thus the remainder is –26, so B.

19. Sketching a graph of the equation yields:  Thus the range is all positive reals, or A.

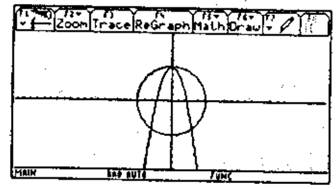
20. 2 miles ÷ 5 mph, $\dfrac{2}{5}hr \bullet \dfrac{60\,\text{min}}{1hr} = 24$ min, Thus D.

21. 6 minutes for 2 cuts = 3 pieces, so 3 minutes per cut. 3 cuts for 4 pieces yields 9 minutes, Thus B.

22. As the log rotates the crab walks the hypotenuse of a triangle with legs of 8 and 6 (2 times 3, circumference). Thus the length walked is $\sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100} = 10$, Thus B.

23. $7^{x+1} = 13$, $\frac{(x+1)(\log 7)}{\log 7} = \frac{\log 13}{\log 7}$, $x = \frac{\log 13}{\log 7} - \frac{\log 7}{\log 7} = \frac{\log 13 = \log 7}{\log 7}$, Thus C.

24. $x^2 + y^2 = 36$ is a circle with center @ (0,0) radius = 6. $x^2 + y = 6$, $y = -x^2 + 6$. $y = (-1)(x-0)^2 + 6$ is a parabola with vertex @ (0.6) facing down with 3 points in common.



25. $p = \pm 1, \pm 2$ $q = \pm 1, \pm 3$. Therefore $\frac{p}{q} = \pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1, \pm 2$, Thus C.

26. $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25} \ldots = \frac{n}{(n+1)^2}$, Thus D.

27. $y = 2(x^2 + 2x + \_\_) + 5$. $y = 2(x^2 + 2x + 1) + 5 - 2$. $y = 2(x+1)^2 + 3$. Parabola vertex (-1,3), Thus C.

28. $(x)(x+4) = 9600$. $x^2 + 4x - 9600 = 0$, $(x-96)(x+100) = 0$, Thus 96 and 100, A.

29. $-5x^2 - 5x + 9 = 0$. $\frac{5 \pm \sqrt{-5^2 - 4(-5)(9)}}{2(-5)} = \frac{5 \pm \sqrt{205}}{-10} = \frac{-1}{2} \pm \frac{\sqrt{41}}{\sqrt{20}}$, Thus B.

30. f(x) = x-4, g(x) –x + 6. g(f(0)) = (0-4) = -4, 4 + 6 = 10 f(g(2)) = -2 + 6 = 4, 4 - 4 = 0. 10-0 = 10, Thus D.