Solutions Algebra II Individual Feb. Regional 2-11-95

1. Since $x^2<x$ for $0<x<1$, $x^2-x<0$ B

2. $|x-3| = \begin{cases} x-3 & x\geq 3 \\ 3-x & x<3 \end{cases}$ C

3. $\sqrt{2}i \cdot \sqrt{8}i = \sqrt{16}i^2 = -4$ B

4. $g(1)=1^2=1$, $f(g(1))=f(1)=3$ A

5. C

6. $\frac{1}{x-1} + \frac{1}{(x-1)(x+1)} + \frac{1}{(x-1)(x^2+x+1)} = \frac{(x+1)(x^2+x+1)}{LCD} + \frac{x^2+x+1}{LCD} + \frac{x+1}{LCD}$. Coefficient of $x^2=3$ D
[LCD $=(x-1)(x+1)(x^2+x+1)$]

7. $15+3i-20i-4i^2 = 19-17i$; $a+b=2$ C

8. $(1+i)^2 = 2i$
So $(1+i)^5 = (1+i)^4(1+i) = (2i)^4(1+i) = 16(1+i)$ D

9. B

10. $(x-4)^2 = 4(x-1)$
$x^2-8x+16 = 4x-4$
$x^2-12x+20=0$
$(x-10)(x-2)=0$
$x=10$, $x=2$ (Not valid)
$x=10$ so $x^2-2x+1=81$ A

11. $(x-y)(x+y)=7$
$(x-y)(-1)=7$
$x-y=-7$
$x=-7+y$ C

12. at $(1,1)$: $1=\log_a 1+d$
$1=d$
at $(2,4)$: $4=\log_a 2+1$
$3=\log_a 2$
$2=a^3$
$1+\sqrt[3]{2}$ E C

13. average rate $=\frac{\text{miles}}{\text{hours}}=\frac{90}{\frac{5}{6}+\frac{1}{2}}$ B
hours $\Rightarrow$ $t\cdot r=d$ $\begin{cases} t\cdot r=d \\ t\cdot 70=50 \\ t\cdot 60=40 \end{cases}$

14. $f(g(x))=g(f(x))=x$ is the def. of an inverse relation. To find $f^{-1}(x)$
$y=2x+1$
$x=2y+1$
$\frac{x-1}{2}=y$ B

15. $(2,0)(4,-1)$ $m=\frac{-1-0}{4-2}=\frac{-1}{2}$
$x+2y=k$
$2+0=k$
$2=k$
$x+2y=2$ A

16. $F = 20+\frac{1}{2}F$
$\frac{1}{2}F=20$
$F=40$ B

17. Parabola with vertex $(0,2)$
$y-2=a(x-0)^2$
$y-2=\frac{-1}{4}(x)^2$
at $y=1$ $1=\frac{1}{4}(x)^2$
$\pm 2 = x$ C

18. B

19. Change of base:
$\frac{\log 81}{\log a}=\frac{\log 27^{1/4}}{\log b}$
$4\log 3\cdot\log b = \log a\cdot\frac{3}{4}\log 3$
$\frac{16}{3}=\frac{\log a}{\log b}$
$\frac{16}{3}=\log_b a$
$b^{16/3}=a$ D

2-11-95

Algebra II Regional

20. $S=\frac{k}{T^2}$, $S_2=\frac{k}{(\frac{1}{2}T)^2}$, $S_2=\frac{4k}{T^2}=\frac{k}{T^2}\cdot 4 = 4S$ A

21. B

22. $t\cdot r=d$
$t\cdot r=5t$ C
$r=5$

23. E all are true

24. $a+b=ab$
$a-ab=-b$
$a(1-b)=-b$
$a=\frac{-b}{1-b}=\frac{b}{b-1}$ D

25. Roots: 2, $-3+i$, $-3-i$
sum $=-6$
product $=9-i^2=10$
$f(x)=(x-2)(x^2+6x+10)$
$f(-1)=-3(1-6+10)=-15$ B

26. Area $=\frac{1}{2}(x)(2^x)=\frac{1}{2}bh$
$=x\cdot 2^{x-1}$ B

27. $\frac{1}{x}+\frac{2}{y}=4$, $\frac{4}{x}+\frac{-3}{y}=1$ } let $a=\frac{1}{x}$, $b=\frac{1}{y}$ } $a+2b=4$, $4a-3b=1$ } $3(a+2b)=12$, $2(4a-3b)=2$
$11a=14$
$a=\frac{14}{11}$ so $x=\frac{11}{14}$
$\frac{14}{11}+2b=4$
$b=\frac{15}{11} \Rightarrow y=\frac{11}{15}$ $\frac{x}{y}=\frac{15}{14}$
A

28. $b^2-4ac<0$
$k^2-4(12)<0$
$k^2<48$
$-\sqrt{48}<k<\sqrt{48}$
D

29. The lines are the hyperbola's asymptotes.
$\frac{x^2}{25}-\frac{y^2}{4}=1$ Lines: $y=\pm\frac{2}{5}x$ D

30. 7, 17, 27, 37, 47, 57, 67, 77, 87, 97 } 20 "7"s
and 70, 71, 72, 73, 74, 75, 76, 78, 79 }
B