5 5

Solutions – Alg 2 Written Test
Feb 20, 1993

1. D $\quad 5 * x = \frac{5^2 - x}{5 - x}$

$11 * 0 = \frac{11^2 - 0}{11 - 0} = 11$

$\frac{25 - x}{5 - x} = 11 \Rightarrow x = 3$

2. D $\quad f(g(h(2x+1)))$

$= f(g((2x+1)^2))$

$= f(g(4x^2+4x+1))$

$= f\left(\frac{4x^2+4x+1-1}{4}\right)$

$= f(x^2+x)$

$= \sqrt{x^2+x}$

3 D $\quad f(x) = \ln x^2 = y$

$e^y = x^2 \Rightarrow e^{y/2} = x = f^{-1}(x)$

$f^{-1}(0) = e^{0/2} = 1 \Rightarrow (0,1)$

4. E $\quad \log_x x = 1$

So, $\cancel{\pm 1 = 2x}$

$\cancel{x = \pm 1/2}$

5. C $\quad$ Either $x^2 - 3 < 1$ or $-(x^2-3) < 1$

$x^2 - 4 < 0 \qquad x^2 - 2 > 0$

$-2 < x < 2$ or $x < -\sqrt{2}$ or $x > \sqrt{2}$

So, $-2 < x < -\sqrt{2}$ or $\sqrt{2} < x < 2$

6. E $\quad$ The inverse is $x = 2y^2$

Using substitution:

$y = 2(2y^2)^2$

$y - 8y^4 = 0$

$y = 0, 1/2$

$x = 0, \pm 1/2$

Eliminate $(-1/2, 1/2)$. It does not satisfy $x = 2y^2$

$\therefore (0,0)$ and $(\frac{1}{2}, \frac{1}{2})$

7. C $\quad$ Test by graphing.

8. D $\quad$ For $x = 2$,

$\sum_{k=0}^{5} (8+k) = P + \sum_{k=0}^{5} k$

$\Rightarrow \sum_{k=0}^{5} 8 + \sum_{k=0}^{5} k = P + \sum_{k=0}^{5} k$

$\therefore P = \sum 8 = 48$

9. $\underline{\underline{B}}$ Let x = amt. removed

$(10-x)(.5) + x(0) = 10(.4)$

$x = 2$

10. $\underline{\underline{C}}$ $f(x) = \dfrac{x+1}{x}$

$f(f(x)) = \dfrac{\frac{x+1}{x}+1}{\frac{x+1}{x}} = \dfrac{2x+1}{x+1}$

$f(f(f(x))) = f\left(\dfrac{2x+1}{x+1}\right) = \dfrac{3x+2}{2x+1}$

11. $\underline{\underline{E}}$ Let $\log_{64} x = w$ and $\log_x 64 = \frac{1}{w}$.

So, $w - \frac{1}{w} = \frac{5}{6}$ and $w = \frac{3}{2}$ or $-\frac{2}{3}$.

If $\log_{64} x = -\frac{2}{3}$, then $x = \frac{1}{16}$

If $\log_{64} x = \frac{3}{2}$, then $x = 512$

12. $\underline{\underline{C}}$ Let P = orig. pop.

r = rate of increase

$P(1+r)(1-r)(1+r)(1-r) =$

$P(1-r^2)^2 = .81P$

$1-r^2 = .9 \Rightarrow r = .316$

The nearest % is 32%

13. $\underline{\underline{A}}$ Rationalizing each denom., we get:

$\dfrac{\sqrt{1}-\sqrt{2}}{-1} + \dfrac{\sqrt{2}-\sqrt{3}}{-1} + \dfrac{\sqrt{3}-\sqrt{4}}{-1} + \ldots + \dfrac{\sqrt{99}-\sqrt{100}}{-1}$

$= (\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + (\sqrt{4}-\sqrt{3}) + \ldots + (\sqrt{100}-\sqrt{99})$

$= \sqrt{100} - \sqrt{1} = 9$

14. $\underline{\underline{A}}$ Let side of the square $= \frac{x}{4}$. Then circumference of circle $= 12-x$ and the radius $= \dfrac{12-x}{2\pi}$

Tot. Area $= \dfrac{x^2}{16} + \pi\left(\dfrac{12-x}{2\pi}\right)^2$

If $x=0$, Area $= \frac{36}{\pi}$

If $x=12$, Area $= 9$

The max. area occurs at an endpt. of the domain since ax^2 is negative.

So, $x = 0$

15. $\underline{\underline{D}}$ $(x^2-3x+5)(x-2)^9 =$

$(x^2-3x+5)\left(x^9 + 9x^8(-2) + \binom{9}{2}x^7(-2)^2 + \ldots\right)$

The coeff. of x^7 will be:

$x^2\left[\binom{9}{4}x^5(-2)^4\right] - 3x\left[\binom{9}{3}x^6(-2)^3\right] + 5\left[\binom{9}{2}x^7(-2)^2\right]$

$= 2016x^7 + 2016x^7 + 720x^7 = 4752x^7$

16. $\underline{\underline{B}}$ $f(3-x) = 3(3-x)^2 + 6(3-x)$

$= 46 - 24x + 3x^2$

The min occurs at $x = \frac{24}{6} = 4$

and $g(4) = -2$

17. C Since the boy runs 4 mph relative to the train, it will take him $\frac{1}{8}$ hr. to reach the front. Relative to the ground, he is running 5 mph. Thus, $5\left(\frac{1}{8}\right) = \frac{5}{8}$

18. C Let w = width of river. When the ferries 1st meet, one has traveled 900 m and the other $(w-900)$ m. At the 2nd mtg., the 1st traveled the remaining $(w-900)+400$ back from the opp. shore. The 2nd has traveled 900 m across the river & meets the 1st after an additional $(w-500)$ for a total of $(w+400)$ m. The ratio of distances traveled is equal so:

$$\frac{900}{w-900} = \frac{w-400}{w+400}$$

and $w=2200$

19. B Since a & b are integers, we estimate btwn 2 & 3 for $\sqrt{1+\sqrt{21+12\sqrt{3}}}$. Thus, the only possibilites are $(1,2)$ and $(1,3)$. Then, plug in to test. OR, $\rightarrow$

$$\sqrt{21+12\sqrt{3}} = \sqrt{(3+2\sqrt{3})^2}$$

and if the orig radical $= x$, we get

$$x^2 = 1+(3+2\sqrt{3}) \text{ and } x = 1+\sqrt{3}$$

So $(a,b) = (1,3)$

20. D
$$\log_{2x} 256 = \frac{\log_x 256}{\log_x 2x}$$

$$= \frac{2\log_x 16}{\log_x 2 + \log_x x} = \frac{2y}{\frac{1}{4}\log_x 16 + 1}$$

$$= \frac{2y}{\frac{1}{4}y+1} = \frac{8y}{y+4}$$

21. D The roots are $r-d, r, r+d$.

Sum $\Rightarrow 3r = 6$

$r=2$ is a root.

Substitution yields $c=64$

22. A
$$\left(\sqrt[5]{\sqrt{18}+\sqrt{2}}\right)^2$$

$$= \sqrt[5]{(\sqrt{18}+\sqrt{2})^2}$$

$$= \sqrt[5]{20+2\sqrt{36}} = \sqrt[5]{32}$$

$$= 2$$

23. B $2\cdot 4^x + 6^x = 9^x$

$2^{2x+1} + 2^x\cdot 3^x - 3^{2x} = 0$

$(2^{x+1} - 3^x)(2^x + 3^x) = 0$

$2^{x+1} = 3^x$ $\quad \cancel{2^x = -3^x} \Rightarrow$ Not poss.

$\left(\frac{2}{3}\right)^x = \frac{1}{2}$

$x = \log_{\frac{2}{3}}\frac{1}{2} \Rightarrow a = \frac{1}{2}$

24. E Since the ratio is constant, the series is geometric,

$S = \frac{\frac{1}{10}}{\frac{3}{10}} = \frac{1}{3}$

25. C Let $x = 5 \Rightarrow 2f(5) + f(-4) = 25$

Let $x = -4 \Rightarrow 2f(-4) + f(5) = 16$

Solving 2 equations of 2 unknowns,

we get $f(5) = \frac{34}{3}$

26. E $a = \#$ 1¢ stamps

$b = \#$ 2¢ stamps

$c = \#$ 3¢ stamps

$a + 2b + 5c = 100$

$a = 10b$

Thus $12b + 5c = 100$

and $c = 20 - \frac{12b}{5}$

So, b is a mult of 5.

If $b \geq 10 \Rightarrow c < 0$.

So, $b = 5$ and $c = 8$

27. A $\left(a + \frac{1}{a}\right)^2 = 3^2$

$a^2 + \frac{1}{a^2} + 2 = 9 \Rightarrow a^2 + \frac{1}{a^2} = 7$

So, $\left(a - \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} - 2 = 7 - 2 = 5$

$\left|a - \frac{1}{a}\right| = \sqrt{5}$

28. A $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$, so $(x+y)^2 = xy$

and $x^2 + xy + y^2 = 0$.

Thus, $\left(\frac{x}{y}\right)^2 + \frac{x}{y} + 1 = 0$

and $\frac{x}{y} = \frac{-1 \pm i\sqrt{3}}{2}$

29. E Let $x = 6^{th}$ term. Then, $1+2+4+8+16+32 = 63$

$63 = \frac{6}{2}(1 + x)$

$x = 20$

30. B The first man occupies the following series of places in successive moves:

1, 3, 5, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 2, 4, 6, 10

Thus, 20 moves