3. $\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & e^2\sqrt{\pi} \end{bmatrix} \bullet \begin{bmatrix} \sin^2(x) & \sin(x)\cos(x) & 1 \\ \cos^2(x) & -\sin(x)\cos(x) & 0 \\ \sqrt{\pi} & \frac{1}{e} & 0 \end{bmatrix} = ?$

A. $\begin{bmatrix} 1 & 0 & 1 \\ \cos(2x) & -\sin(2x) & -1 \\ \pi e^2 & e\sqrt{\pi} & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2\sin(x)\cos(x) & -1 \\ \pi e & e\sqrt{\pi} & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 1 \\ \cos(2x) & \sin(x)\cos(x) & 1 \\ \pi e^2 & e+\sqrt{\pi} & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 1 \\ \sin^2(x) - \cos^2(x) & \sin(2x) & 0 \\ \pi e^2 & e\sqrt{\pi} & 0 \end{bmatrix}$

E. NOTA

4. Find the sum of the following two vectors $\vec{u}$ & $\vec{v}$; $\vec{u} + \vec{v}$

$$\vec{u} = [1, 0, 1]\ ;\ \vec{v} = [-1, 1, -1]$$

A. $[0, 0, 0]$ B. $[1, 1, 1]$ C. $[1, 0, -1]$ D. $[0, 1, 0]$ E. NOTA

5. Find $\frac{\vec{w}}{2}$ where $\vec{w}$ is the resulting vector after the linear transformation **A** is applied to the vector $\vec{u}$:

$$[A\vec{u} = \vec{w}],\ \vec{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix},\ A = \begin{bmatrix} 1 & 1 & 2 \\ -2 & -3 & 3 \\ 0 & -4 & 2 \end{bmatrix}$$

A. $\frac{\vec{w}}{2} = \begin{bmatrix} 1/2 \\ -2 \\ 3 \end{bmatrix}$ B. $\frac{\vec{w}}{2} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ C. $\frac{\vec{w}}{2} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ D. $\frac{\vec{w}}{2} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ E. NOTA

6. Let $\Omega = \begin{bmatrix} -1 & 2 & -3 & 4 \\ -4 & 3 & 2 & -1 \\ 1 & -3 & -2 & 4 \\ 4 & -2 & 3 & -1 \end{bmatrix}$. Find $-\left(\Omega^t\right)$. (Note Ω^t denotes the transpose of Ω)

A. $-\left(\Omega^t\right) = \begin{bmatrix} -1 & -4 & 1 & 4 \\ 2 & 3 & -3 & -2 \\ -3 & 2 & -2 & 3 \\ 4 & -1 & 4 & -1 \end{bmatrix}$

B. $-\left(\Omega^t\right) = \begin{bmatrix} 1 & 4 & -1 & -4 \\ -2 & -3 & 3 & 2 \\ 3 & -2 & 2 & -3 \\ -4 & 1 & -4 & 1 \end{bmatrix}$

C. $-\left(\Omega^t\right) = \begin{bmatrix} -4 & 2 & -3 & 1 \\ -1 & 3 & 2 & -4 \\ 4 & -3 & -2 & 1 \\ 1 & -2 & 3 & -4 \end{bmatrix}$

D. $-\left(\Omega^t\right) = \begin{bmatrix} -4 & 3 & -2 & 1 \\ 1 & -2 & -3 & 4 \\ -4 & 2 & 3 & -1 \\ 1 & -3 & 2 & -4 \end{bmatrix}$

E. NOTA

7. $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$. Evaluate: $4\left(A^t\right)^2 + B^2 - 4(A)B$

A. $\begin{bmatrix} 1 & 0 \\ 3 & -3 \end{bmatrix}$ B. $\begin{bmatrix} 5 & -4 \\ -6 & 9 \end{bmatrix}$ C. $\begin{bmatrix} 5 & 4 \\ 4 & -7 \end{bmatrix}$ D. $\begin{bmatrix} 9 & 0 \\ 8 & -7 \end{bmatrix}$ E. NOTA

8. Find the inverse of $\begin{bmatrix} e & \pi \\ \sqrt{2} & 1 \end{bmatrix}$.

A. $\begin{bmatrix} 1 & \pi \\ \sqrt{2} & e \end{bmatrix}$

B. $\begin{bmatrix} -1 & \pi \\ \sqrt{2} & -e \end{bmatrix}$

C. $\dfrac{1}{\pi\sqrt{2} - e}\begin{bmatrix} -1 & \pi \\ \sqrt{2} & -e \end{bmatrix}$

D. $\dfrac{1}{\pi\sqrt{2}}\begin{bmatrix} e & -\pi \\ -\sqrt{2} & 1 \end{bmatrix}$

E. NOTA

9. Find the approximate value for $\det(\Delta)$, which is in a 10% neighborhood of the actual solution.

$$\Delta = \begin{bmatrix} -3 & \pi & \frac{-1}{2.7182} \\ \frac{7}{22} & 1 & 0 \\ \pi & e & e^{i\pi} \end{bmatrix}$$

A. $4 + \frac{\pi + e}{\pi e}$ B. $1 + \frac{\pi^2 + e}{e}$ C. $4 + \frac{\pi^2 - e}{\pi e}$ D. $4 + \frac{\pi - e}{\pi e}$ E. NOTA

10. Which of the following is/are true statements about the properties of the determinants A and B?

I. $\det(AB) = \det(A)\det(B)$, where A and B are square matrices (n x n).
II. $\det(A+B) = \det(A) + \det(B)$, where A and B are square matrices (n x n).
III. $\det(A^t) = -\det(A)$, where A is an n x n square matrix

A. Only **I** B. Only **I & II** C. Only **I, II, & III** D. Only **II & III** E. NOTA

11. Given $A\vec{x} + \vec{b} = \vec{c}$, $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, & $\vec{c} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$. Solve for $\vec{x}$.

A. $\vec{x} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$ B. $\vec{x} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$ C. $\vec{x} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ D. $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ E. NOTA

12. Find the solution set to the following system of equations:

$$\begin{cases} x_1 + 3x_3 = 2 \\ x_2 + x_3 = 1 \\ x_4 = -1 \end{cases}$$

Note: **R** represents the real numbers

A. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \end{bmatrix} - \alpha \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} \middle| \alpha \in \mathsf{R} \right\}$

C. $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \alpha \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \end{bmatrix} \middle| \alpha \in \mathsf{R} \right\}$

E. NOTA

13. Tommy was using Cramer's Rule to solve the following system of equations:

$$\begin{cases} x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + x_3 = 5 \\ -2x_1 + 2x_2 - x_3 = -8 \end{cases}$$

As Professor O'Dailey was grading Tommy's homework, she noticed that Tommy didn't finish solving the system. His partial answer was $x_1 = \frac{?}{3}$, $x_2 = \frac{?}{3}$ & $x_3 = \frac{?}{3}$. Complete Tommy's work and find the value for x_2.

A. $x_2 = 1$ B. $x_2 = -3$ C. $x_2 = -1$ D. $x_2 = 3$ E. NOTA

14. Consider the vector space R^3/R (real valued 3-tuples with real coefficients). Given two different basis $U=[u_1,u_2,u_3]$ & $V=[v_1,v_2,v_3]$, find the linear map **T** that takes vectors

$$\alpha=\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix}=\alpha_1 u_1+\alpha_2 u_2+\alpha_3 u_3 \text{ to vectors } \beta=\begin{bmatrix}\beta_1\\\beta_2\\\beta_3\end{bmatrix}=\beta_1 v_1+\beta_2 v_2+\beta_3 v_3.$$

$T:U\to V$, where α and β are equivalent in R^3/R.

$$u_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, u_2=\begin{bmatrix}1\\0\\1\end{bmatrix}, u_3=\begin{bmatrix}0\\1\\1\end{bmatrix} \quad \& \quad v_1=\begin{bmatrix}1\\2\\3\end{bmatrix}, v_2=\begin{bmatrix}1\\3\\2\end{bmatrix}, v_3=\begin{bmatrix}3\\2\\1\end{bmatrix}$$

A. $T=\begin{bmatrix}0&2&4\\1&-1&-1\\2&1&2\end{bmatrix}$ B. $T=\begin{bmatrix}0&2&4\\1&-1&-1\\2&1&-2\end{bmatrix}$ C. $T=\begin{bmatrix}1&-1&0\\-1&0&2\\1&1&1\end{bmatrix}^{-1}$ D. $T=\begin{bmatrix}0&1&2\\1&0&1\\2&3&-2\end{bmatrix}$ E. NOTA

15. Find the inverse of $\begin{bmatrix}1&0&0&0&0\\0&1&1&0&0\\0&-1&-1&0&0\\0&0&0&3&0\\0&0&0&0&2\end{bmatrix}$.

A. $\begin{bmatrix}1&0&0&0&0\\0&-1&-1&0&0\\0&1&1&0&0\\0&0&0&.\bar{3}&0\\0&0&0&0&.5\end{bmatrix}$ B. $\begin{bmatrix}1&0&0&0&0\\0&1&-1&0&0\\0&-1&1&0&0\\0&0&0&3&0\\0&0&0&0&2\end{bmatrix}$ C. $\begin{bmatrix}1&0&0&0&0\\0&-1&-1&0&0\\0&0&0&0&0\\0&0&0&-.\bar{3}&0\\0&0&0&0&.5\end{bmatrix}$

D. $\begin{bmatrix}1&0&0&0&0\\0&1&1&0&0\\0&1&1&0&0\\0&0&0&3&0\\0&0&0&0&2\end{bmatrix}$ E. NOTA

16. Solve the following equations for $[x_1, x_2, x_3, x_4]$

$$3x_1 - 4x_2 = 1 \qquad 3x_3 - 4x_4 = 3$$
$$-2x_1 + 5x_2 = 2 \qquad -2x_3 + 5x_4 = 4$$

A. $[x_1, x_2, x_3, x_4] = \left[\frac{13}{7}, \frac{8}{7}, \frac{31}{7}, \frac{18}{7}\right]$

B. $[x_1, x_2, x_3, x_4] = \frac{1}{7}[13, -8, 31, 18]$

C. $[x_1, x_2, x_3, x_4] = [31, 18, -13, 8]$

D. $[x_1, x_2, x_3, x_4] = \left[\frac{-13}{7}, \frac{8}{7}, \frac{31}{7}, \frac{-18}{7}\right]$

E. NOTA

17. Consider a unit square (square with side length = 1) in R^2, centered around the origin (0,0). Create vectors $v_1, \ldots, v_4$ by connecting the center of the square to each vertex, i.e. $v_1 = \left[\frac{1}{2}, \frac{1}{2}\right], v_2 = \left[\frac{-1}{2}, \frac{1}{2}\right], \ldots$ Find the matrix ϖ that will produce the vectors $v_1', \ldots, v_4'$ formed from the above unit square rotated 195° counter-clockwise, $\varpi(v_1)^t = \left(v_1'\right)^t$. (Note: $\sqrt{2+\sqrt{3}} = \frac{\sqrt{2}}{2}\left(\sqrt{3}+1\right)$ and $\sqrt{2-\sqrt{3}} = \frac{\sqrt{2}}{2}\left(\sqrt{3}-1\right)$)

A. $\varpi = \sqrt{2}\begin{bmatrix} -1-\sqrt{3} & 1-\sqrt{3} \\ \sqrt{3}-1 & -\sqrt{3}-1 \end{bmatrix}$

B. $\varpi = \frac{-1}{2}\begin{bmatrix} \sqrt{2+\sqrt{3}} & \sqrt{\sqrt{3}-2} \\ \sqrt{2-\sqrt{3}} & \sqrt{2+\sqrt{3}} \end{bmatrix}$

C. $\varpi = \frac{1}{2}\begin{bmatrix} -\sqrt{2+\sqrt{3}} & \sqrt{2-\sqrt{3}} \\ -\sqrt{2-\sqrt{3}} & -\sqrt{2+\sqrt{3}} \end{bmatrix}$

D. $\varpi = \frac{1}{\sqrt{2}}\begin{bmatrix} 1+\sqrt{3} & \sqrt{3}-1 \\ 1-\sqrt{3} & \sqrt{3}+1 \end{bmatrix}$

E. NOTA

18. Solve for **x**: $\begin{vmatrix} x^2 & 3\pi & 0 \\ 0 & ex & -\pi \\ -e^2 & \frac{1}{\pi} & \frac{1}{e} \end{vmatrix} = \pi^2 e^2 + x^3$

A. $x = \sqrt{2}e\pi$

B. $x = \pm\sqrt{2}ei\pi$

C. $x = \mp\sqrt{2}e\pi$

D. $x = \pm i\pi$

E. NOTA

19. How many of the following definitions are true? Let **A** be a 2 x 2 matrix.

I. $\text{trace}(\mathbf{A}) = a_{11} + a_{22}$

II. Let $B = A^t, \left[a_{ji}\right]$

III. $\det(\mathbf{A}) = a_{22}a_{21} - a_{11}a_{12}$

IV. $A^{-1} = \frac{1}{\det(A)}\text{adj}(A)$

A. 0 B. 1 C. 2 D. 3 E. NOTA

20. One day, while doing her linear algebra homework, Laura rotated her paper 90° clockwise. She toyed with the idea of physically rotating matrices by multiplying it with a "rotator matrix," i.e. 2 x 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ rotated 90° clockwise becomes $\begin{bmatrix} c & a \\ d & b \end{bmatrix}$ (take $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ with the "rotator matrix" being $\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$).
She became obsessed with finding these "rotator matrices" (**W** a "rotator matrix" for a 90° clockwise rotation of **A**, **WA** = **B, B** the rotated version of **A**). She suddenly became stuck finding a 90° counter-clockwise "rotator matrix" for $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Which matrix would solve Laura's dilemma?

A. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ C. $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ D. $\begin{bmatrix} -1 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$ E. NOTA

21. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$. Evaluate $\frac{A}{\det(A)}\left(adj(A) + 5\mathbf{I} \right)$. (Note: adj(**A**) stands for adjoint matrix & **I** is the 3 x 3 identity matrix)

A. $\begin{bmatrix} 7 & 1 & -1 \\ -2 & 6 & 2 \\ 1 & 1 & 6 \end{bmatrix}$ B. $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ E. NOTA

22. $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & \cdots & \cdots \\ 1 & 2 & 3 & 4 & 5 & \cdots & \cdots \\ 1 & 3 & 6 & 10 & \cdots & \cdots \\ 1 & 4 & 10 & \cdots & \cdots \\ 1 & 5 & \cdots & \cdots \\ 1 & \cdots & \cdots \\ \vdots \end{bmatrix}$. What is a_{ij}, for i, j > 1? (Let 0! = 1)

A. $\frac{(i+j-2)!}{(i+1)!(j+1)!}$ B. $\frac{i!}{(i+j-1)!(j-1)!}$ C. $\frac{(i+j)!}{i!\,j!}$ D. $\frac{i!}{j!}$ E. NOTA

23. Consider a triangle ΔABC in R^3. **A** = [1,1,1] is a point on the plane P_1, spanned by vectors [1,1,0] & [0,1,1]. **B** = [-1,0,2] is a point on the plane P_2, whose normal is $\overrightarrow{AB}$. Let $C = P_1 \cap P_2 \cap (xy-\text{plane})$. Find the area of ΔABC.

A. $\frac{\sqrt{6}}{2}$ B. $2\sqrt{3}$ C. $\sqrt{3}$ D. $\frac{\sqrt{2}}{2}+1$ E. NOTA

24. $\begin{vmatrix} i & 0 & 1+i \\ 4-i & 2+3i & \sqrt{2}-i \\ i\sqrt{2} & 1 & 1+3i \end{vmatrix} = ?$

A. $(5\sqrt{2}-5)-4i$ B. $4+i$ C. $3i\sqrt{2}-4$ D. $3i\sqrt{2}$ E. NOTA

25. Which of the following matrices forms a representation for $z = a + bi \in C$ (where **C** is the set of complex numbers), i.e. given two such matrices **Z** & **W**, they preserve complex addition and multiplication?

A. $\begin{bmatrix} a & b \\ a & b \end{bmatrix}$ B. $\begin{bmatrix} a & -b \\ -b & a \end{bmatrix}$ C. $\begin{bmatrix} -a & b \\ b & b \end{bmatrix}$ D. $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ E. NOTA

26. Find $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}^n$, where n>1.

A. $\begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$, $\theta = \tan^{-1}(2)$

B. $5\begin{bmatrix} 2n & n \\ -n & 2n \end{bmatrix}$

C. $\left(\sqrt{5}\right)^n \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix}$, $\theta = \cos^{-1}\left(\frac{2\sqrt{5}}{5}\right)$

D. $\left(\sqrt{5}\right)^n \begin{bmatrix} \cos\left(\frac{2n\sqrt{5}}{5}\right) & \sin\left(\frac{n\sqrt{5}}{5}\right) \\ -\sin\left(\frac{n\sqrt{5}}{5}\right) & \cos\left(\frac{2n\sqrt{5}}{5}\right) \end{bmatrix}$

E. NOTA

27. One of the more interesting sequences in mathematics is the Fibonacci Sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

The recursion formula of this sequence is $x_n = x_{n-1} + x_{n-2}$, $x_0 = x_1 = 1$. Find the matrix equation which relates $X_n = \begin{bmatrix} x_n \\ x_{n-1} \end{bmatrix}$ to $X_{n-1} = \begin{bmatrix} x_{n-1} \\ x_{n-2} \end{bmatrix}$, where x_n represents the n+1 term in the Fibonacci Sequence.

A. $X_{n-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} X_n$ B. $X_n = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} X_{n-1}$ C. $X_{n-1} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} X_n$ D. $X_n = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} X_{n-1}$ E. NOTA

28. Solve for x: $\begin{vmatrix} x & 2 \\ 0.5 & 3x+4 \end{vmatrix} = 3x + 1$

A. $\left\{-1, \frac{2}{3}\right\}$ B. $\left\{-1, -\frac{2}{3}\right\}$ C. $\left\{1, -\frac{2}{3}\right\}$ D. $\left\{1, \frac{2}{3}\right\}$ E. NOTA

29. Evaluate: $\begin{bmatrix} 5 & -4 & 0 \\ -3 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}^{-1}$

A. $\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{3}{2} & -\frac{5}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

B. $\begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ \frac{3}{2} & -\frac{5}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

C. $\begin{bmatrix} -1 & -2 & 0 \\ -\frac{3}{2} & -\frac{5}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

D. $\begin{bmatrix} -\frac{5}{2} & -\frac{1}{2} & 0 \\ -3 & 2 & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

E. NOTA

30. Solve: $\begin{cases} 0.4000x_1 + 561.6x_2 = 562.0 \\ 73.03x_1 - 43.03x_2 = 30.00 \end{cases}$

A. $[x_1, x_2] = [0.0, 1.001]$

B. $[x_1, x_2] = [0.0, 1.0]$

C. $[x_1, x_2] = [-.001, 1.001]$

D. $[x_1, x_2] = [1.0, 1.0]$

E. NOTA