

# MU ALPHA CONVENTION 1991

## MATRICES AND DETERMINANTS TOPIC TEST

1. Solve the inequality:  $\begin{vmatrix} x & x-2 \\ 5 & 10 \end{vmatrix} < 0$

- A.  $x < -3$    B.  $x < -2$    C.  $x > 2$    D.  $x > 3$    E. none of these

2. Compute:  $\begin{vmatrix} \sqrt{2} & 0 & 0 & 0 \\ 6 & -\sqrt{2} & 0 & 0 \\ 8 & 3 & -1 & 0 \\ 2 & 5 & 6 & 7 \end{vmatrix}$

- A. -14   B. 2   C. -2   D. 0   E. 14

3. If  $\begin{vmatrix} W & X \\ Y & Z \end{vmatrix} = 4$ , then  $\begin{vmatrix} 4W & 4X \\ 4Y & 4Z \end{vmatrix} =$

- A. 4   B. 8   C. 16   D. 32   E. 64

4. Find  $f(A)$  if  $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$  and  $f(t) = t^2 - 3t + 7I$ , where  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ .

- A.  $\begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$    B.  $\begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$    C.  $\begin{bmatrix} -3 & 12 \\ -6 & 9 \end{bmatrix}$    D.  $\begin{bmatrix} 3 & -12 \\ 6 & -9 \end{bmatrix}$   
 E. none of these

5. If  $A = [2 \ 1]$  and  $B = \begin{bmatrix} 1 & -2 & 0 \\ 4 & 5 & -3 \end{bmatrix}$ , then find  $AB$ .

- A.  $AB$  is not defined   B.  $[6 \ 1 \ -3]$    C.  $\begin{bmatrix} 6 \\ 1 \\ -3 \end{bmatrix}$    D.  $[6 \ 3 \ -3]$   
 E. none of these

6. If  $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$  then  $A^{-2} =$

- A.  $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ 1 & 1 \end{bmatrix}$     B.  $\begin{bmatrix} \frac{3}{4} & -\frac{9}{4} \\ \frac{5}{2} & \frac{9}{2} \end{bmatrix}$     C.  $\begin{bmatrix} \frac{3}{4} & -\frac{5}{2} \\ -\frac{5}{4} & \frac{9}{2} \end{bmatrix}$     D.  $\begin{bmatrix} 1 & 1 \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$   
 E. none of these

7. Find the area of the triangle with vertices  $A(2, -4)$ ,  $B(6, 7)$ , and  $C(-5, -3)$ .

- A. 20    B.  $\frac{41}{2}$     C. 41    D.  $\frac{81}{2}$     E. 81

8. Solve for  $X$ :  $\begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix} \cdot X + \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ .

- A.  $\begin{bmatrix} 17 & -41 \\ 41 & 97 \end{bmatrix}$     B.  $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$     C.  $\begin{bmatrix} 3 & -7 \\ 3 & -7 \end{bmatrix}$     D.  $\begin{bmatrix} -1 & -1 \\ -3 & -3 \end{bmatrix}$   
 E.  $\begin{bmatrix} -19 & -23 \\ -49 & -61 \end{bmatrix}$

9. The image of  $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$  under the mapping  $\begin{bmatrix} 1 & 4 & 1 \\ -2 & 0 & 0 \\ 3 & 2 & -3 \end{bmatrix}$  is:

- A.  $\begin{bmatrix} 9 \\ -6 \\ 5 \end{bmatrix}$     B.  $[9 \ -6 \ 5]$     C.  $\begin{bmatrix} 7 \\ 16 \ -3 \end{bmatrix}$     D.  $[7 \ 16 \ -3]$   
 E. none of these

10. The solution for the variable "y" is given using Cramer's rule to solve a 3x3 system of equations with unknowns (x,y,z). If the solution for the variable "z" is written using Cramer's rule, the numerator would be:

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & -3 \\ 1 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3 & -3 \\ 1 & -1 & 1 \end{vmatrix}}$$

- A.  $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3 & 2 \\ 1 & -1 & 0 \end{vmatrix}$       B.  $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix}$       C.  $\begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{vmatrix}$       D.  $\begin{vmatrix} 1 & -1 & 1 \\ 3 & 3 & -3 \\ 1 & 1 & 1 \end{vmatrix}$   
 E. none of these

11. Which one of the following statements about determinants is not true?

- A. The determinant of a matrix equals the determinant of its transpose.  
 B. Interchanging any two rows or columns will affect the sign of the determinant, but not the absolute value of the determinant.  
 C. Multiplying a single row or column of an nxn matrix by a scalar s will cause the value of the determinant to be multiplied by  $s^n$ .  
 D. Addition or subtraction of a nonzero multiple of any row or column to or from another row or column does not change the value of the determinant.  
 E. If all elements of a row or column in a square matrix equal zero, the determinant will equal zero.

12. Find the cofactor of the element in row 3 column 2 of matrix A

if  $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ -2 & 2 & 3 \end{bmatrix}$ .

- A. -10      B. -5      C. -1      D. 1      E. 10

13. Find the sum of the elements in the first column of matrix A if

$$A = \begin{bmatrix} 5 & -3 & 1 \\ 2 & 1 & 4 \\ 3 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}.$$

- A. 3    B. 9    C. 25    D. 27    E. 30

14. Find the determinant of the inverse of matrix  $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ .

- A.  $\frac{1}{32}$     B.  $\frac{1}{16}$     C.  $\frac{1}{2}$     D. 8    E. 16

15. The rank of matrix  $A = \begin{bmatrix} 3 & 1 & -2 & 4 \\ 2 & 0 & -5 & 1 \\ 1 & -1 & 2 & 6 \end{bmatrix}$  is:

- A. 0    B. 1    C. 2    D. 3    E. 4

16. Find the sum of the entries in the multiplicative inverse of  $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$ .

- A. -4    B. 0    C. 2    D. 4    E. 8

17. The matrix  $A = \begin{bmatrix} 1 & 2 & -5 \\ -4 & 1 & -6 \\ 6 & 3 & -4 \end{bmatrix}$  in row-reduced echelon form is:

- A.  $\begin{bmatrix} 1 & 0 & \frac{7}{9} \\ 0 & 1 & -\frac{26}{9} \\ 0 & 0 & 0 \end{bmatrix}$     B.  $\begin{bmatrix} 1 & 0 & -\frac{7}{9} \\ 0 & 1 & \frac{26}{9} \\ 0 & 0 & 0 \end{bmatrix}$     C.  $\begin{bmatrix} 1 & 0 & \frac{7}{9} \\ 0 & 1 & -\frac{26}{9} \\ 0 & 0 & 1 \end{bmatrix}$     D.  $\begin{bmatrix} 1 & 0 & \frac{5}{9} \\ 0 & 1 & -\frac{13}{9} \\ 0 & 0 & 1 \end{bmatrix}$   
E. none of these

18. The determinant  $\begin{vmatrix} x & y & 1 \\ x^2 & y^2 & 1 \\ x^3 & y^3 & 1 \end{vmatrix}$  is equal to:

- A.  $xy(x+1)(y-1)(x-y)$     B.  $xy(x-1)(y+1)(y-x)$   
 C.  $xy(x-1)(y-1)(y-x)$     D.  $xy(x-1)(y+1)(x-y)$   
 E.  $xy(x-1)(y+1)(x-y)$

19. Given the system of equations  $\begin{cases} x+y-z+w=3 \\ y+z+w=0 \\ -z-w=4 \\ 2x-y+w=-3 \end{cases}$ . If the determinant of the coefficient matrix is 3, use Cramer's Rule to solve for  $x$ .

- A. -7    B.  $-\frac{11}{3}$     C.  $\frac{7}{3}$     D.  $\frac{13}{3}$     E. 5

20. The equation of the plane through the points  $P(-1,2,3)$ ,  $Q(-3,1,2)$ , and  $R(-5,4,6)$  is  $x+Ay+Bz+C=0$ . Find  $A+B+C$ .

- A. -5    B. -3    C. 1    D. 3    E. 5

21. An equation of a circle passing through the points  $(-3,1)$ ,  $(2,4)$ , and  $(5,-2)$  is given by the equation:

A.  $\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 10 & -3 & 1 & 1 \\ 20 & 2 & 4 & 1 \\ 29 & 5 & -2 & 1 \end{vmatrix} = 0$     B.  $\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 10 & -3 & 1 & 1 \\ 20 & 2 & 4 & 1 \\ 29 & 5 & -2 & 1 \end{vmatrix} = 1$

C.  $\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 10 & 1 & -3 & 1 \\ 20 & 4 & 2 & 1 \\ 29 & -2 & 5 & 1 \end{vmatrix} = 1$     D.  $\begin{vmatrix} x^2+y^2 & x & y \\ 10 & -3 & 1 \\ 20 & 2 & 3 \\ 29 & 5 & -2 \end{vmatrix} = 1$     E. none of these

22. Find the volume of the tetrahedron with vertices  $A(0,-1,3)$ ,  $B(3,2,1)$ ,  $C(1,-1,2)$ , and  $D(2,3,1)$ .

- A.  $\frac{1}{2}$     B.  $\frac{2}{3}$     C. 1    D. 2    E. 3

23. Which of the matrices is (are) singular? ( $x, y \neq 0$ )

I.  $\begin{bmatrix} x & y \\ x & y \end{bmatrix}$     II.  $\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$     III.  $\begin{bmatrix} x & -y \\ x & y \end{bmatrix}$     IV.  $\begin{bmatrix} -x & y \\ -x & -y \end{bmatrix}$     V.  $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

- A. III, IV    B. I, II    C. I    D. II, V  
 E. all 5 are singular

24. Find the matrix that will rotate  $P(x, y)$  through an angle of  $60^\circ$  about the origin.

A.  $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$     B.  $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$     C.  $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$     D.  $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

- E. none of these

25. Find the eigenvalue(s) of  $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ .

- A. 1;3    B. 2;4    C. -1;3    D. 3    E. 4