Matrices and Determinants
Topic Test Solutions

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① $10x - 5x + 10 < 0$

$5x < -10$

$x < -2$

ans B

② $\begin{vmatrix} \sqrt{2} & 0 & 0 & 0 \\ 6 & -\sqrt{2} & 0 & 0 \\ 8 & 3 & -1 & 0 \\ 2 & 2 & 6 & 7 \end{vmatrix}$

ans. E

$(\sqrt{2})(-\sqrt{2})(-1)(7) = 14$

③ $\begin{vmatrix} 4w & 4x \\ 4y & 4z \end{vmatrix} = 4 \cdot 4 \cdot 4$

ans E

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④ $f(A) = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}^2 - 3 \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A

$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - \begin{bmatrix} 3 & -6 \\ 12 & 15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$

⑤ $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 0 \\ 4 & 5 & -3 \end{bmatrix} =$

B

$\begin{bmatrix} 6 & 1 & -3 \end{bmatrix}$

⑥ $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -2 \\ -1 & 4 \end{bmatrix}$ ans. C

$A^{-2} = \frac{1}{4}\begin{bmatrix} 3 & -10 \\ -5 & 18 \end{bmatrix}$

$A^{-2} = \begin{bmatrix} \frac{3}{4} & -\frac{5}{2} \\ -\frac{5}{4} & \frac{9}{2} \end{bmatrix}$

⑦ $A(2,-4)$ $B(6,7)$ $C(-5-3)$ ans D

$P_1(4,11)$ $P_2(-7,1)$

$\frac{1}{2}\begin{vmatrix} 4 & 11 \\ -7 & 1 \end{vmatrix} = \frac{1}{2}(4+77)$

$= \frac{81}{2}$

⑧ $\begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix} \cdot X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ans B

$9X = 0$ either

$X = 0$ B C, D

$\frac{1}{9}\begin{bmatrix} 1 & 2 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix} X = \frac{1}{9}\begin{bmatrix} 1 & 2 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

⑨ $\underset{3\times 3}{\begin{bmatrix} 1 & 4 & 1 \\ 2 & 0 & 0 \\ 3 & 2 & -3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}} = \begin{bmatrix} 9 \\ -6 \\ 5 \end{bmatrix}$ ans A

(10) To solve for z by Cramer's Rule the x and y columns remain the coefficients. The z column is replaced by the constants. Read these from the given

ans A

$$z_n = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 3 & 2 \\ 1 & -1 & 0 \end{vmatrix}$$

(11) A. T B. T C. F D. T E. T ans C

Should be just 5

(12)

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ -2 & 2 & 3 \end{bmatrix}$$

ans B

$$\text{cofactor}_{32} = -\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$
$$= -5$$

(13)

$$A = \begin{bmatrix} 5 & -3 & 1 \\ 2 & 1 & 4 \\ 3 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

ans D

Find $a_{11} + a_{21} + a_{31}$

$$11 + 8 + 8 = 27$$

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(14) $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ ans B

$$\det \text{ of } A^{-1} = \frac{1}{\det . \text{ of } A}$$
$$= \frac{1}{6+2+0-0-0+8}$$
$$= \frac{1}{16}$$

(15) $\begin{bmatrix} 3 & 1 & -2 & 4 \\ 2 & 0 & -5 & 1 \\ 1 & -1 & 2 & 6 \end{bmatrix}$ ans. D

can find 3×3 det.

$\therefore$ Rank 3

(16) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$ ans C

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -4 & -1 & 2 \\ 2 & 0 & -2 \\ -2 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & \frac{1}{2} & -1 \\ -1 & 0 & 1 \\ 1 & \frac{1}{2} & -1 \end{bmatrix}$$

add entries: $2+1-1=2$

(17)

$$A = \begin{bmatrix} 1 & 2 & -5 \\ -4 & 1 & -6 \\ 6 & 3 & -4 \end{bmatrix}$$

ans a

add R_2 to R_3

$$\begin{bmatrix} 1 & 2 & -5 \\ -4 & 1 & -6 \\ 2 & 4 & -10 \end{bmatrix}$$

Divide R_3 by 2

$$\begin{bmatrix} 1 & 2 & -5 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{bmatrix}$$

Multiply R_1 by (-1) and add to R_3

$$\begin{bmatrix} 1 & 2 & -5 \\ -4 & 1 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

multiply R_1 by 4 + add to R_2

$$\begin{bmatrix} 1 & 2 & -5 \\ 0 & 9 & -26 \\ 0 & 0 & 0 \end{bmatrix}$$

Divide R_2 by 9

$$\begin{bmatrix} 1 & 2 & -5 \\ 0 & 1 & -\frac{26}{9} \\ 0 & 0 & 0 \end{bmatrix}$$

Multiply R_2 by (-2) + add to R_1

$$\begin{bmatrix} 1 & 0 & \frac{7}{9} \\ 0 & 1 & -\frac{26}{9} \\ 0 & 0 & 0 \end{bmatrix}$$

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(18)

$$\begin{vmatrix} x & y & 1 \\ x^2 & y^2 & 1 \\ x^3 & y^3 & 1 \end{vmatrix}$$

$$\begin{vmatrix} x & y & 1 \\ x^2-x & y^2-y & 0 \\ x^3-x & y^3-y & 0 \end{vmatrix}$$

ans C

Evaluate by minors

$$+1\begin{vmatrix} x^2-x & y^2-y \\ x^3-x & y^3-y \end{vmatrix}$$

$$xy\begin{vmatrix} x-1 & y-1 \\ x^2-1 & y^2-1 \end{vmatrix}$$

$$xy(x-1)(y-1)\begin{vmatrix} 1 & 1 \\ x+1 & y+1 \end{vmatrix}$$

$$xy(x-1)(y-1)[(y+1)-(x+1)]$$

$$xy(x-1)(y-1)(y-x)$$

(19)

$$x = \frac{\begin{vmatrix} 3 & 1 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 4 & 0 & -1 & -1 \\ -3 & -1 & 0 & 1 \end{vmatrix}}{3}$$

Most students probably will not use Cramer's Rule.

Ans C

$$x = \frac{\begin{vmatrix} 0 & 0 & -1 & 2 \\ 0 & 1 & 1 & 1 \\ 4 & 1 & 0 & 0 \\ -3 & -1 & 0 & 1 \end{vmatrix}}{3}$$

$y = 4$

$$\begin{cases} x - z + w = -1 \\ z + w = -4 \\ 2x + w = 1 \end{cases}$$

$$\begin{cases} x + 2w = -5 \\ 2x + w = 1 \end{cases}$$

$$\begin{cases} x + 2w = -5 \\ 4x + 2w = 2 \end{cases}$$

$3x = 7$

$x = \frac{7}{3}$

$$x = \frac{\begin{vmatrix} \cancel{0} & \cancel{0} & \cancel{-1} & \cancel{2} \\ 0 & 1 & 0 & 3 \\ 4 & 1 & 0 & 0 \\ -3 & -1 & 0 & 1 \end{vmatrix}}{3}$$

$$x = \frac{-1 \begin{vmatrix} 0 & 1 & 3 \\ 4 & 1 & 0 \\ -3 & -1 & 1 \end{vmatrix}}{3} = \frac{-1(-12-4+9)}{3}$$

$$= \frac{7}{3}$$

(20) $P(-1, 2, 3)$ $Q(-3, 1, 2)$ $R(-5, 4, 6)$

$\overrightarrow{PQ}(-2, -1, -1)$ $\overrightarrow{PR}(-4, 2, 3)$

Use Cross Products

x	y	z
-2	-1	-1
-4	2	3

Ans A

$\overrightarrow{PQ} \times \overrightarrow{PR} = (-1, 10, -8)$

$-x + 10y - 8z = 1 + 20 - 24$

$-x + 10y - 8z = -3$

$x - 10y + 8z - 3 = 0$

$A + B + C = -10 + 8 - 3$

$= -5$

(21) Formula for Cartesian equation of the circle through 3 noncollinear pts. Ans. A

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Thus answer is A.

(22) ans B

$$\frac{1}{6}\begin{vmatrix} 0 & -1 & 3 & 1 \\ 3 & 2 & 1 & 1 \\ 1 & -1 & 2 & 1 \\ 2 & 3 & 1 & 1 \end{vmatrix} =$$

$$\frac{1}{6}\begin{vmatrix} 0 & -1 & 3 & 1 \\ 3 & 3 & -2 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 4 & -2 & 0 \end{vmatrix} =$$

$$2\cdot\frac{1}{6}\cdot 1\begin{vmatrix} 3 & 3 & -2 \\ 1 & 0 & -1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$\frac{2}{6}\left(0-\cancel{3}-4-0+6+\cancel{3}\right)$$

$$\frac{4}{6} = \frac{2}{3}$$

(23) A singular matrix is one whose determinant is 0. Thus I is the only one. ans c

(24) The transformation that rotates through an angle θ about the origin is:

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Thus when $\theta = 60^\circ$

$$A = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

(25) Characteristic Matrix:

$$\begin{bmatrix} 1-X & 2 \\ 2 & 1-X \end{bmatrix}$$

Ch. Polynomial: $(1-X)(1-X)-4=0$

ch. Equation $X^2-2X-3=0 \Rightarrow$

3, -1 are Eigenvalues