Mu Alpha Theta National Convention 2004
Theta Logs, Exponents, Radicals
Answers

#	Answer	#	Answer
1	C	18	C
2	C	19	B
3	B	20	B
4	D	21	A
5	B	22	B
6	D	23	C
7	B	24	A
8	D	25	D
9	D	26	C
10	E	27	B or E
11	A	28	D
12	C	29	B
13	B	30	D or E
14	D	TB1	$2^{b\pm\sqrt{b^2-4ac}}$
15	B	TB2	$\frac{-5-\sqrt{3}}{6}$
16	C	TB3	$\frac{1}{x^2y^2z^2}$
17	D		

1. Answer: C

$\log_2 x = 2\log_4 x$

$$\frac{\frac{\log x}{\log 2}}{\frac{\log x}{\log 4}} = 2 \qquad \frac{\log 4}{\log 2} = 2$$

2 = 2 true for $x > 0$

3. Answer: C

$\log_2(\log_9 3) = \log_x 7$

$$\log_2 \frac{1}{2} = \log_x 7$$

$-1 = \log_x 7 \qquad x = \frac{1}{7}$

3. Answer: B

$$\frac{\left(16x^5\right)^{\frac{1}{4}}}{4\,x^{\frac{1}{4}}} = \frac{1}{4}\left(\frac{16x^5}{x}\right)^{\frac{1}{4}} = \frac{1}{4}\left(16x^4\right)^{\frac{1}{4}} =$$

$$\frac{1}{4}(2x) = \frac{x}{2}$$

4. Answer: D

$$\left(\sqrt[3]{4c^2d^3}\right)^3\left(\sqrt{2}cd\right)^2 = 4c^6d^9 \cdot 2c^2d^2 = 8c^8d^{11}$$

5. Answer: B

Given x, y > 0, $\log_y x + \log_x y = \frac{10}{3}$ and

xy = 144. Let $v = \log_y x$, then $\frac{1}{v} = \log_x y$, solve $v + \frac{1}{v} - \frac{10}{3} = 0$ to find $v = \log_y x = 3$ or $v = \frac{1}{3}$.

Without loss of generality, assume
$x > y$. Then $\log_y x = 3$, $x = y^3$ →

$xy = y^4 = 144$, so that $y = \sqrt{12} = 2\sqrt{3}$; $\sqrt{12}x = 144 \quad x = 12\sqrt{12}$ → $\frac{2\sqrt{3} + 24\sqrt{3}}{2} = 13\sqrt{3}$

6. Answer: D

Since $a \neq 0$, the only x for which $f(x) = -\sqrt{2}$ is x = 0. Since

$f(f(\sqrt{2})) = -\sqrt{2}$, $f(\sqrt{2})$ must be 0. so $2a - \sqrt{2} = 0$, or $a = \frac{\sqrt{2}}{2}$

7. Answer: B

$$\frac{2^{n+4} - 2^{n+1}}{2^{n+4}} = 1 - 2^{-3} = \frac{7}{8}$$

8. Answer: D

$$\sqrt{x\sqrt{x\sqrt{x}}} = \left(x\left(x(x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(x\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$\left(x^{\frac{7}{4}}\right)^{\frac{1}{2}} = x^{\frac{7}{8}} = \sqrt[8]{x^7}$$

9. Answer: D
Point (x,y) is on the graph of G` iff the point (y, -x) is on the graph of G, so
–x = log y. This last equation is equivalent to $y = 10^{-x}$, which is an equation for G`. Since (x, y) = (10, 1) is on G, it follows that (x, y) = (-1, 10) must be on G`, which shows that no other choice is correct.

10. Answer: E $a^{2\log_a 7} + 3\log_a\left(a^2\right) = a^{\log_a 49} + \log_a(a^6)$

$49 + 6 = 55$

11. Answer: A
$(4^{-1} - 3^{-1})^{-1} = \left(\frac{1}{4} - \frac{1}{3}\right)^{-1} = \left(\frac{-1}{12}\right)^{-1} = -12$

12. Answer: C
$x + \sqrt{x^2 - 1} + \frac{1}{x - \sqrt{x^2 - 1}} = 20$ then

$x^2 - x^2 + 1 + 1 = 20x - 20\sqrt{x^2 - 1}$

$1 - 10x = -10\sqrt{x^2 - 1}$

$1 - 20x + 100x^2 = 100x^2 - 100$

$x = \frac{101}{20}$

$x^2 + \sqrt{x^4 - 1} + \frac{1}{x^2 + \sqrt{x^4 - 1}} =$

$x^2 + \sqrt{x^4 - 1} + x^2 - \sqrt{x^4 - 1} =$

$2x^2$

$2\left(\frac{101}{20}\right)^2 = 51.005$

13. Answer: B
$\log\left(\sqrt[4]{\frac{x^2}{y^8 z^6}}\right) = \frac{1}{2}\log x - 2\log y - \frac{3}{2}\log z$

17. Answer: D

If x > y > 0, then $\frac{x^y y^x}{y^y x^x} = \frac{x^{y-x}}{y^{y-x}} = \left(\frac{x}{y}\right)^{y-x}$

18. Answer: B

$6^6 + 6^6 + 6^6 + 6^6 + 6^6 + 6^6 = 6(6^6) = 6^7$

16. Answer: C

$$\frac{a+b^{-1}}{a^{-1}+b} = 13$$

$$\frac{ab(a+b^{-1})}{ab(a^{-1}+b)} = \frac{a(ab+1)}{b(1+ab)} = \frac{a}{b} = 13$$

Thus a = 13b, and a + b $\leq$ 100 implies 14b $\leq$ 100 b $\leq \frac{50}{7}$ so 0< b $\leq$ 7 For each of the seven possible values of b = 1, 2, 3, 4, 5, 6, 7, the pair (13b, b) is a solution.

17. Answer: D

$1^{-1} - (-1)^2 + 2^1 = 1 - 1 + 2 = 2$

18. Answer: C

$$\sqrt[4]{y} + \sqrt[4]{625y} = 2(1+\sqrt[4]{y})$$

$$6\sqrt[4]{y} = 2 + 2\sqrt[4]{y}$$

$$\sqrt[4]{y} = \frac{1}{2} \quad y = \frac{1}{16}$$

24. Answer: B

$$f(x) = \sqrt{8x - x^2} - \sqrt{14x - x^2 - 48}$$

f(x) = $\sqrt{16-(x-4)^2} - \sqrt{1-(x-7)^2}$; The first expression is the y-coordinate for the upper half of the circle with C(4,0) r = 4; the second expression is the y-coordinate of the upper half of the circle with C(7, 0), r = 1. f(x) is real valued only when 6 $\leq$ x $\leq$ 8, and maximum value will be attained when x = 6. f(6) = $\sqrt{16-(6-4)^2} - \sqrt{1-(6-7)^2} = 2\sqrt{3}$

25. Answer: B

Solving $10z^2 - 3iz - k = 0$, where z is a complex variable and $i^2 = -1$.

$x = \frac{3i \pm \sqrt{-9+40k}}{20}$ which has a discriminant if k = 1 of 31, so A is false. If k is a negative real number, then D is a negative real number, so B is true. If k = I, D = -9 + 40i = 16 + 40i – 25 = $(4 + 5i)^2$ and the roots are $x = \frac{1}{5} + \frac{2}{5}i$ and $x = \frac{-1}{5} - \frac{1}{10}i$ so C is false. If k=0 which is a complex number, then the roots are 0 and $\frac{3}{10}i$, so D is false.

26. Answer: A

$\log_2(\log_2(\log_2(x))) = 2$, $\log_2(\log_2(x))=4$,
$\log_2(x)=16$, $x = 2^{16} = 65536$

27. Answer: B

Recall log (ab) = log a + log b, i.e. a product = a sum (logs)

23. Answer: C

$$5\log_4 x + 2\log_4(x+3) = \log_4\left(x^5\right) + \log_4\left[(x+3)^2\right] = \log_4\left[x^5(x+3)^2\right]$$

24. Answer: A

$i^{42} + i^{30} + i^{27} - i^{13} = -2 - 2i$

25. Answer: D

$\sqrt{x+14} = x - 16$; $x + 14 = x^2 - 32x + 256$;
$x^2 - 33x + 242 = 0$; $(x-11)(x-22)=0$;
x=11, or x = 22, reject x = 11

26. Answer: C

$(\sqrt{-4})(i^{27}) - (i^5)(3i^{10}) = 2+3i$

27. Answer: B

$\sqrt{x^2 - 7x - 8}$ is defined when
$x^2 - 7x - 8 \geq 0$; $(x-8)(x+1) \geq 0$

$(-\infty, -1] \cup [8, \infty)$

30. Answer: D

$z = a + bi \rightarrow |z| = \sqrt{a^2+b^2} \rightarrow$
$|z|^2 = a^2 + b^2$ If $z + |z| = 2 + 8i$, then
$a + bi + \sqrt{a^2+b^2} = 2 + 8i \rightarrow b = 8$
$a + \sqrt{a^2+b^2} = 2$
$a^2 + b^2 = 4 - 4a + a^2$
$4a = 60 \rightarrow a = 15$
$|z|^2 = a^2 + b^2$ $|z|^2 = 225 + 64 = 289$

31. Answer: B

sin x = 3 cos x, then tan x = 3
$\sin x = \frac{3}{\sqrt{10}}$ and $\cos x = \frac{1}{\sqrt{10}} \rightarrow$
$\sin x \cos x = \left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) = \frac{3}{10}$

30. Answer: D

$A = Pe^{rt}$ $5P = Pe^{.06t}$ $\ln 5 = 0.06t$
$t = 26.82$

TIE BREAKERS

3. Answer: $2^{b\pm\sqrt{b^2-4ac}}$

$(\log_4 x)^2 - \log_4 x^b - a = 0$

$(\log_4 x)^2 - b\log_4 x - a = 0$

$$\log_4 x = \frac{b \pm \sqrt{b^2 - 4ac}}{2}$$

$$x = 2^{2\left(\frac{b\pm\sqrt{b^2-4ac}}{2}\right)} = 2^{b\pm\sqrt{b^2-4ac}}$$

4. Answer: $\frac{-5-\sqrt{3}}{6}$

$$\frac{4\sqrt{50} - 3\sqrt{98} + \sqrt{24}}{2\sqrt{32} - 5\sqrt{2} - \sqrt{54}} = \frac{20\sqrt{2} - 21\sqrt{2} + 2\sqrt{6}}{8\sqrt{2} - 5\sqrt{2} - 3\sqrt{6}}$$

$$\frac{-\sqrt{2} + 2\sqrt{6}}{3\sqrt{2} - 3\sqrt{6}} \cdot \frac{3\sqrt{2} + 3\sqrt{6}}{3\sqrt{2} + 3\sqrt{6}} = \frac{-5 - \sqrt{3}}{6}$$

3. Answer: $\frac{1}{x^2y^2z^2}$

$(x + y + z)^{-1}(x^{-1} + y^{-1} + z^{-1})(xy + yz + zx)^{-1}[(xy)^{-1} + (yz)^{-1} + (zx)^{-1}]$

$$\frac{1}{x+y+z} \cdot \frac{yz + xz + xy}{xyz} \cdot (xy + xz + yz)^{-1} \cdot \frac{z + x + y}{xyz}$$

$$\frac{1}{x^2y^2z^2}$$