Logarithms and Exponents-Theta
2000 National Mu Alpha Theta Convention

1) Find all values of x such that $\log_2 x^2 = 4$?

A) 4 B) 2 C) 4,-4 D) 2,-2 E) NOTA

2) Evaluate: $\log_7 5 - \log_7 (5/7)$.

A) -1 B) 2 C) 1 D 0 E) NOTA

3) For what positive values of x and y does $(\log_x y)(\log_y x) = 1$?

A) $x > 1$ B) $y > 1$ C) $x,y > 1$ D) all positive values of x and y E) NOTA

4) Evaluate $\log_{\sqrt{8}} \sqrt[3]{16}$.

A) 2 B) 9/8 C) 8/9 D) ½ E) NOTA

5) Find z if $\log_z 3 - \log_z 9 + \log_z 1/3 = 4$.

A) $\sqrt{3}/3$ B) $\sqrt{3}$ C) 1/3 D) 3 E) NOTA

6) For how many positive integers n is the value of $\log_{2/3} n$ an integer?

A) 1 B) 2 C) infinitely many D) none E) NOTA

7) Given that a is positive, simplify: $\dfrac{\log a^2}{\log a}$.

A) a B) 2 C) $\log a$ D) a^2 E) NOTA

8) Find x: $\log_{4/25} x = -3/2$.

A) 8/125 B) 125/8 C) 625/16 D) 16/625 E) NOTA

9) Between what two consecutive integers is $\left(1 + \dfrac{1}{10^{100}}\right)^{10^{100}}$?

A) 2 and 3 B) 1 and 2 C) 3 and 4 D) 99 and 100 E) NOTA

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10) At how many points do the graphs of $y = \log_2 x$ and $x = \log_3 y$ intersect?

A) 0 B) 1 C) 2 D) infinitely many E) NOTA

11) Given positive reals a and b such that $\log_a b + \log_b a = 2$, which of the following must be true?

A) $a > b$ B) either $a = 2b$ or $b = 2a$ C) $a = b$ D) either $a = b^2$ or $b = a^2$
E) NOTA

12) Given that $\log_{10} 4 = x$, which of the following is equal to $\log_{10} 5$?

A) $1 - x$ B) $1 - 2x$ C) $2x - 1$ D) $1 - x/2$ E) NOTA

13) If $\log_a b = 64$, find $\log_{a^2} b^3$.

A) 96 B) 16 C) 512 D) 128/3 E) NOTA

14) Find all y such that $\log_y (y + 30) = 2$.

A) -5 B) -5,6 C) 1/6 D) 6 E) NOTA

15) Find the sum of all y such that $\dfrac{3^{2y+1} + 3^2}{3^y} = 28$.

A) 1 B) 3 C) 0 D) -1 E) NOTA

16) Order the following from greatest to least: $x = 2^{2^{2^2}}$, $y = 3^{3^{3}}$, $z = 4^{4^4}$.

A) z, y, x B) y, x, z C) x, y, z D) x, z, y E) NOTA

17) Evaluate $\dfrac{(\log_2 4)(\log_4 8)(\log_8 16)\ldots(\log_{2048} 4096)}{(\log_3 9)(\log_9 27)(\log_{27} 81)\ldots(\log_{2187} 6561)}$.

A) 1 B) 3/2 C) 5/3 D) 4/3 E) NOTA

18) The first (not final!) three digits of 11^{15} are ABC. The last three digits of 11^{15} are DEF. Find $A+B+C+D+E+F$.

A) 20 B) 24 C) 27 D) 31 E) NOTA

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19) Given that $\log_2 5 = 2.322$, what is the smallest positive integer n such that 2^n has 20 digits when written in base 5?

A) 47 B) 46 C) 45 D) 44 E) NOTA

20) Evaluate: $3^{\log_9 3^{500}}$.

A) 3^{500} B) 3^{750} C) 3^{1000} D) 3^{250} E) NOTA

21) Find the sum of all values of x which satisfy $2^{x+4} - 4^x = 63$.

A) 4 B) $\log_2 63$ C) 2 D) 16 E) NOTA

22) Which of the following 3 savings accounts should I prefer? Assume that all 3 pay interest annually and all interest is reinvested in the account.

I) Pays 4% annual interest for 2 years, then 6% annual interest for 2 years.
II) Pays 6% annual interest for 2 years, then 4% annual interest for 2 years.
III) Pays 5% annual interest for 4 years.

A) I B) II C) III D) They are all equivalent E) NOTA

23) Given that $\log_{10} 2 \approx 0.3010$, how many digits are in 25^{52} in base 10?

A) 70 B) 71 C) 72 D) 73 E) NOTA

24) Order the following from least to greatest: $x = 10^{20^{30}}$, $y = 30^{20^{10}}$, $z = 10^{30^{20}}$, $w = 20^{30^{10}}$.

A) y, w, z, x B) w, y, x, z C) x, z, w, y D) z, x, y, w E) NOTA

25) At how many points do the graphs of $y = \log_4 x$ and $y = \log_8 x\sqrt{x}$ coincide?

A) None B) 1 C) 2 D) infinitely many E) NOTA

26) For how many positive integers n is $6^n < 5^n + 4^n$?

A) 1 B) 2 C) 3 D) infinitely many E) NOTA

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27) What integer is closest to $\sum_{i=1}^{10} \log_{720} i$?

A) 1 B) 2 C) 3 D) 4 E) NOTA

28) Evaluate $\left\lfloor \frac{1}{\log_{2^{1000}} \pi^{1000}} + \frac{1}{\log_{5^{1000}} \pi^{1000}} \right\rfloor$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

A) 0 B) 2 C) 3 D) 10 E) NOTA

29) If, for some pair of positive real numbers x and y, we have $\log_x y > x$, then which of the following must be true?

A) $y > 1$ B) $y < x$ C) $x > 1$ D) $y > x$ E) NOTA

30) Find the greatest integer n such that $59^n + 60^n > 61^n$.

A) 28 B) 29 C) 30 D) 32 E) NOTA