

1999 Mu Alpha Theta National Convention
Logarithms & Exponents & Radicals
Theta Division

1. If $\log \tan x = m$, then $\log \cot x$ is equivalent to what?

- A) $\frac{1}{m}$ B) $-m$ C) $1 - m$ D) $\frac{-1}{m}$ E) NOTA

2. $(2^x)(4^x) =$ what?

- A) 8^x B) 8^{2x} C) 2^{x^2} D) 4^{3x} E) NOTA

3. When simplified, $(x^{-1} + y^{-1})^{-1}$ is equal to what?

- A) $x + y$ B) $\frac{xy}{x + y}$ C) xy D) $\frac{1}{xy}$ E) NOTA

4. The graph of $y = 2^x$ lies in :

- A) the first quadrant only B) the first and second quadrants C) the first and third quadrants
D) the first and fourth quadrants E) NOTA

5. If $y = 2^n$, by what number is y multiplied when n is increased by 2?

- A) 2 B) 4 C) $n + 2$ D) y^2 E) NOTA

6. The square root of the square root of the square root of a number is 3, then the number is:

- A) 3^6 B) $\sqrt[8]{3}$ C) 3^8 D) 3^4 E) NOTA

7. Simplify: $[(4)^{\frac{-3}{2}} \sqrt{2^{12}} + (\frac{1}{8})^{\frac{-2}{3}}] \div (27)^{\frac{1}{3}}$

- A) $\frac{2}{3}$ B) 4 C) $\frac{4}{3}$ D) $\frac{10}{3}$ E) NOTA

8. Solve: $\sqrt[4]{y} + \sqrt[4]{625y} = 2(1 + \sqrt[4]{y})$

- A) $\frac{16}{2401}$ B) $\frac{16}{343}$ C) $\frac{1}{16}$ D) $\frac{16}{256}$ E) NOTA

9. Simplify: $3 + \frac{1}{1 + \frac{1}{3 + \sqrt{15}}}$

- A) 1 B) 3 C) $\sqrt{15}$ D) $3 + \sqrt{15}$ E) NOTA

10. Simplify: $\sqrt{600} + \sqrt{864} - \sqrt{\frac{243}{3}}$

- A) $12\sqrt{6} - 9$ B) $22\sqrt{6} - 9$ C) $14\sqrt{6} - 21\sqrt{3}$ D) $12\sqrt{3} - 6$ E) NOTA

11. The conjugate of $\sqrt[3]{a} - \sqrt[3]{b}$ is:

- A) $\sqrt[3]{a^2} + \sqrt[3]{b^2}$ B) $\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}$ C) $\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}$
 D) $\sqrt[3]{a^2} - \sqrt[3]{b^2}$ E) NOTA

12. Simplify: $\sqrt{\sqrt[3]{\sqrt[4]{x^3}}}$

- A) $\sqrt{x^2}$ B) $\sqrt[4]{x}$ C) $\sqrt[9]{x}$ D) $\sqrt[8]{x}$ E) NOTA

13. If $f(x) = \frac{1}{x} - x$, then find $f(3 + \sqrt{5})$.

- A) $\frac{3 - 3\sqrt{5}}{2}$ B) $\frac{-9 - 5\sqrt{5}}{4}$ C) $\frac{6 - 3\sqrt{5}}{2}$ D) $\frac{3 + 3\sqrt{5}}{2}$ E) NOTA

14. The expression $\sqrt{8 + \sqrt{8 + \sqrt{8 + \sqrt{8 + \dots}}}}$ where the dots indicate an infinite repetition of the pattern can be expressed in the form $\frac{a + \sqrt{b}}{c}$. Find $a + b + c$.

- A) 10 B) 28 C) 36 D) 41 E) NOTA

15. Consider the equation $\log(x + \pi) = \log x + \log \pi$, where x is a positive real number. This equation has:

- A) no solution B) exactly 1 solution C) exactly 2 solution
 D) more the 2, but finitely many solutions E) NOTA

16. Let $T = \frac{1}{3 - \sqrt{8}} - \frac{1}{\sqrt{8} - \sqrt{7}} + \frac{1}{\sqrt{7} - \sqrt{6}} - \frac{1}{\sqrt{6} - \sqrt{5}} + \frac{1}{\sqrt{5} - 2}$ then

- A) $T < -1$ B) $T = 1$ C) $1 < T < 2$ D) $T > 2$ E) NOTA

17. Which positive numbers x satisfy the equation $(\log_3 x)(\log_x 5) = \log_3 5$?

- A) 3 and 5 only B) 3, 5, and 15 only C) all positive $x \neq 1$
 D) only numbers of the form $5^n \cdot 3^m$, where n and m are positive integers E) NOTA

18. The product of $\sqrt[3]{4}$ and $\sqrt[4]{8}$ equals

- A) $\sqrt[7]{12}$ B) $2(\sqrt[7]{12})$ C) $\sqrt[7]{32}$ D) $2(\sqrt[12]{32})$ E) NOTA

19. Find the product of $-\sqrt{16} \cdot \sqrt{-4} \cdot \sqrt{-9}$

- A) -24 B) 24 C) -20i D) -10 E) NOTA

20. If $\sqrt{x+2} = 2$ then $(x+2)^2$ equals

- A) $\sqrt{2}$ B) 4 C) 8 D) 16 E) NOTA

21. For all positive numbers x distinct from 1, $\frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x}$

- A) $\frac{1}{\log_{60} x}$ B) $\frac{1}{\log_x 60}$ C) $\frac{1}{(\log_3 x)(\log_4 x)(\log_5 x)}$
 D) $\frac{12}{(\log_3 x) + (\log_4 x) + (\log_5 x)}$ E) NOTA

22. The fraction $\frac{2(\sqrt{2} + \sqrt{6})}{3\sqrt{2 + \sqrt{3}}}$ is equal to

- A) $\frac{2\sqrt{2}}{3}$ B) $\frac{2\sqrt{3}}{3}$ C) $\frac{4}{3}$ D) $\frac{16}{9}$ E) NOTA

23. Which of the following (where all exponents are integers) is NOT true?

- A) $(-\frac{2}{p})^3 = -\frac{8}{p^3}$ B) $(\frac{x^{-2}y^{-3}}{w^{-4}z^{-5}})^{-1} = \frac{x^2y^3}{w^4z^5}$ C) $(3x)^2(2x^3) = 18x^5$
 D) $(x^{-1} + 2y)^{-1} = x + \frac{1}{2y}$ E) NOTA

24. Which of the following is not ALWAYS true?

A) $\sqrt{x^2 y^6} = xy^3$ B) $(\sqrt[5]{6\sqrt{2}})^{30} = 2$ C) $\sqrt[5]{128} = 2\sqrt[5]{4}$ D) $\sqrt[6]{x^4} \sqrt[6]{x^4} = x^{\frac{4}{3}}$ E) NOTA

25. Determine the value(s) of x in the expression: $\log_x x^{x^2} + \log_x x^{-5x} = \log_x \frac{1}{x^6}$

A) $x = 2, x = 1$ B) $x = 2, x = 3$ C) $x = 4$ D) $x = 3, x = 1$ E) NOTA

26. Simplify: $\frac{3^{x-3}(3^{2x} - 3^{2x-4})}{27^{x-1}}$

A) $\frac{80}{81}$ B) $-\frac{1}{81}$ C) -4 D) 4 E) NOTA

27. Simplify: $\sqrt{5 + 2\sqrt{6}}$

A) $\sqrt{5} + \sqrt{6}$ B) $\sqrt{6} + 1$ C) 5 D) $\sqrt{3} + \sqrt{2}$ E) NOTA

28. If $\log 36 = a$ and $\log 125 = b$, express $\log\left(\frac{1}{12}\right)$ in terms of a and b .

A) $\frac{3b + 2a - 12}{6}$ B) $\frac{3a - 2b + 6}{6}$ C) $\frac{6}{2b - 3a - 6}$ D) $\frac{2b - 3a - 6}{6}$ E) NOTA

29. Find the fourth term of the expansion of $(x \log_8 2 - y \log_3 9)^5$

A) $-80(2^{2x})(y^3)$ B) $-10(x \log_8 4)(\log_3 729)$ C) $\frac{-80}{9}x^2 y^3$
D) $-720x^2 y^3$ E) NOTA

30. Simplify: $\sqrt[3]{\frac{\sqrt{112} - \sqrt{9072}}{\sqrt{112}}} - \sqrt[4]{\frac{\sqrt{1008} - \sqrt{448}}{\sqrt{112}}}$

A) -4 B) -3 C) -2 D) -1 E) NOTA

T1. Find the sum of the roots of the equation: $8e^x + \frac{27}{e^x} = 35$

T2. In interval notation, state the domain of $f(x) = \log_3|x|$?

T3. Find the value of $\frac{x}{y}$ if $\frac{3}{\sqrt{y}} - \frac{1}{\sqrt{x}} = \frac{2}{\sqrt{x} + \sqrt{y}}$