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SOLUTION PAGE

DIVISION: THETA

TEST: Logs and Exponents

①  $\log x = 1.36175$   
 $10^{1.36175} = 23.001...$   
 $\approx 23$  ans: (d)

②  $\log_6 6^{4/3}$   $6^x = 6^{4/3}$   
 $x = 4/3$  ans: (e)

③  $y = 8^x$   $y = \log_2 x$   
 $y = 8^0 = 1$   $0 = \log_2 1$   $\sqrt{1^2 + 1^2} = \sqrt{2}$   
 $(0, 1)$   $(1, 0)$  ans: (a)

④  $16 - x^2 > 0$   
 $(4-x)(4+x) > 0$   
 $-4 < x < 4$  ans: (e)  
 $-3, -2, -1, 0, 1, 2, 3$

⑤  $2^{6 \log_2 3}$   
 $2^{\log_2 3^6} = 3^6 = 729$  ans: (c)

⑥  $\frac{1}{2}^x = 8$   $(2^{-1})^x = 2^3$   
 $-x = 3$   $x = -3$   
ans: (a)

⑦  $A = Pe^{rt}$  (compounded continuously)  
 $A - 1000 = Pe^{0.0575 \cdot 3}$   
 $P = \frac{1000}{e^{0.1725}} = 841.558...$   
 $\approx \$842$   
ans: (a)

⑧  $y' = 9 - 4x$   
 $(4 + 2y)' = 3x$   
 $4x + y = 9 \Rightarrow -8x + -2y = -18$   
 $-3x + 2y = -4$   
 $-11x + 0 = -22$   
 $x = 2$   
 $4(2) + y = 9$   
 $y = 1$   $(2, 1)$  ans: (b)

⑨  $\frac{1}{\log_4 6} = \log_6 9$   $\log_6 9 + \log_6 24$   
 $\log_6 9 \cdot 24$   
 $\log_6 216 = 3$   
ans: (c)

⑩  $\log_3 |x|$   
defined for all real #'s except 0  
ans: (d)

⑪ Middle term will be the 4th term  
 $n = 6, r = 3$   
 $\frac{6!}{3!3!} (\log_2 81)^3 (\log_3 32)^3$   
 $20 \left(\frac{\log_2 81}{\log_2}\right)^3 \left(\frac{\log_3 32}{\log_3}\right)^3$   
 $20 \left(\frac{4 \log_2 3}{\log_2} \cdot \frac{5 \log_3 2}{\log_3}\right)^3$   
 $20 (20)^3 = 20^4$  ans: (b)

⑫  $\frac{\log_3 500}{\log_3 10} = \frac{\log_3 5^3 \cdot 2^2}{\log_3 2 \cdot 5} = \frac{3 \log_3 5 + 2 \log_3 2}{\log_3 2 + \log_3 5}$   
 $\frac{3b + 2a}{a + b} = \frac{2a + 3b}{a + b}$  ans: (c)

⑬  $\frac{3^{3x-3} - 3^{3x-7}}{3^{3x-3}} = 1 - 3^{-4}$   
 $1 - \frac{1}{81} = \frac{80}{81}$  ans: (c)

⑭  $\log 5^{2x+1} = \log 6^{x-2}$   
 $(2x+1) \log 5 = (x-2) \log 6$   
 $2x \log 5 - x \log 6 = -\log 5 - 2 \log 6$   
 $x = \frac{-\log 5 - 2 \log 6}{\log 5 - \log 6} = -3.6388$  ans: (a)

⑮  $10^0 = \log_3 (\log_6 x)$   
 $3^1 = \log_6 x$   
 $x = 6^3$   
 $x = 216$  ans: (c)

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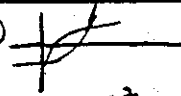
16.  $5^x - 5^{-x} = 6$   
 $5^x(5^x - 5^{-x}) = 6(5^x)$   
 $5^{2x} - 1 = 6(5^x)$   
 $5^{2x} - 6(5^x) - 1 = 0$   
 $5^x = \frac{6 \pm \sqrt{36 - 4(-1)(-1)}}{2(1)}$   
 $5^x = 3 \pm \sqrt{10}$  reject  $3 - \sqrt{10}$   
 $5^x = 3 + \sqrt{10} \rightarrow \log_5 3 + \sqrt{10}$   
 ans: (C)

17.  $\frac{\log 2^{12}}{\log 6^3} \cdot \frac{\log 6^5}{\log 3} \cdot \frac{\log 5^6}{\log 5^5} \cdot \frac{\log 3^5}{\log 2^5}$   
 $\frac{12 \log 2}{3 \log 6} \cdot \frac{5 \log 6}{\log 3} \cdot \frac{6 \log 5}{5 \log 5} \cdot \frac{5 \log 3}{5 \log 2}$   
 $24 \cdot 2^3 \cdot 3^1 \cdot (3+1)(1+1)$   
 $4 \cdot 2 = 8$   
 Answer: (A)

18.  $2 \log_7 a + \frac{1}{2} \log_3 a$   
 $2 \cdot \frac{1}{\log_7 a} + \frac{1}{2} \frac{1}{\log_a 3}$   
 $2 \cdot \frac{1}{.8} + \frac{1}{2} \cdot \frac{1}{.5}$   
 $2.5 + 1 = 3.5$  ans: (B)

19.  $3^x = \frac{1}{10,000}$   
 $x = -8.38$   
 $[x] = [-8.38] = -9$  ans: (A)

20.  $8e^{2x} - 35e^x + 27 = 0$   
 $e^x = \frac{35 \pm \sqrt{(-35)^2 - 4(8)(27)}}{16}$   
 $e^x = \frac{35 \pm 19}{16}$   
 $e^x = \frac{27}{8} \quad e^x = 1$   
 $x = \ln \frac{27}{8} \quad x = \ln 1$   
 $\ln \frac{27}{8} + 0 = \ln \frac{27}{8}$  ans: (B)

21.  intersect in 2 points  
 ans: (C)

22.  $(\frac{3}{5})^{x^2} = (\frac{5}{3})^{x-12}$   $x^2 + 2x - 24 = 0$   
 $(\frac{3}{5})^{x^2} = (\frac{3}{5})^{-2x+24}$   $(x-4)(x+6)$   
 $(\frac{3}{5})^{x^2} = (\frac{3}{5})^{-2x+24}$   $(4, -6)$   
 $x^2 = -2x + 24$  ans: (E)

23.  $\log_5 (x-2)^{1/2} = \log_5 \frac{8}{(x-2)^{3/2}}$   
 $(x-2)^{3/2} (x-2)^{1/2} = 8$   
 $(x-2)^2 = 8$   $x-2 = \pm \sqrt{8}$   
 $x = 2 \pm 2\sqrt{2}$  reject  $2 - 2\sqrt{2}$   
 $2 + 2\sqrt{2}$  ans: (B)


24.  $\log 2^8 \cdot 3^4 \cdot 5^2 \cdot 7$   
 $8 \log 2 + 4 \log 3 + 2 \log 5 + \log 7$   
 $8 \log 2 + 4 \log 3 + 2(1 - \log 2) + \log 7$   
 $6 \log 2 + 4 \log 3 + \log 7 + 2$   
 $6a + 4b + c + 2$  ans: (C)

25.  $\log x^{(\log x)^{1/2}} = \log 10^8$   
 $(\log x)^{1/2} \log x = 8$   
 $(\log x)^{3/2} = 8$   
 $\log x = 8^{2/3} \Rightarrow \log x = 4$   
 $10^4 = 10,000$  ans: (B)

26.  $9 \cdot 9^{1/3} \cdot 9^{1/4} \Rightarrow 9^{1+1/3+1/4} \dots$   
 $9^{7/12} = 27$  ans: (D)  $\frac{1}{1-1/3} = \frac{3}{2}$

27.  $(\log_2 \sqrt{3})(\log_{27} \frac{1}{3}) - (\log_3 \sqrt{5})(\log_{25} 9)$   
 $\frac{1}{2} \log_2 3 \cdot \frac{-1}{3 \log 3} - \frac{1}{2} \log_3 5 \cdot \frac{2 \log 3}{2 \log 5}$   
 $-\frac{1}{2} - -\frac{1}{2} = 0$  ans: (B)

28.  $\log \frac{x^{1/2}}{y^2 z^3} = \frac{1}{2} \log x - (\log y^2 + \log z^3) = \frac{1}{2} \log x - 2 \log y - 3 \log z$  ans: (B)

29.  ans: (C)

30.  $2^{2(x^2+3)} = 2^4$   $2x^2 + 6 = 4$   $x = \pm i$   
 $2x^2 = -2$   $x = \pm i$  ans: (D)