

1992 National Mu Alpha Theta Convention

Logarithms & Exponents Topic Test Answers:

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|-------|-------|
| 1. A | 16. E |
| 2. B | 17. B |
| 3. B | 18. C |
| 4. B | 19. A |
| 5. C | 20. D |
| 6. E | 21. A |
| 7. D | 22. A |
| 8. A | 23. E |
| 9. C | 24. C |
| 10. D | 25. D |
| 11. E | 26. A |
| 12. A | 27. C |
| 13. A | 28. B |
| 14. D | 29. C |
| 15. D | 30. A |

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① $\log_2 8 = x \quad 2^x = 8 \quad 2^x = 2^3 \quad x = 3 \quad A$

② $a = \log_8 225 = \frac{\log 225}{\log 8} = \frac{2 \log 5}{3 \log 2} = \frac{2}{3} \log_2 5 =$

$\frac{2}{3} b \quad B$

③ $(-i)^{3895} = -(i)^{3895} = -(i)^3 = -(-i) = i \quad B$

④ $f(a) = \log_a a = \frac{\log a}{\log a} = 1$

$f(1) = \log_a 1 = 0 \quad (a^0 = 1)$

$f(0) = \log_a 0$ is not defined B

⑤ $4^x = 17$ in log form is $\log_4 17 = x$

$\frac{\log 17}{\log 4} = \frac{\log 17}{2 \log 2} = \frac{1}{2} (\log 17) / (\log 2) \quad C$

⑥ $\log(x^3 - y^3) - \log(x - y) = \log\left(\frac{x^3 - y^3}{x - y}\right) =$

$\log\left(\frac{(x - y)(x^2 + xy + y^2)}{x - y}\right) = \log(x^2 + xy + y^2) \quad E$

⑦ a) $-\left(\frac{8}{27}\right)^{-2/3} = -4/3 \quad \frac{1}{3}$ away from -1

b) $-\left(\frac{8}{27}\right)^{2/3} = -3/4 \quad \frac{1}{4}$ away

c) $-\left(\frac{16}{25}\right)^{-1/2} = -5/4 \quad \frac{1}{4}$ away

d) $-\left(\frac{16}{25}\right)^{1/2} = -4/5 \quad \frac{1}{5}$ away D

⑧ $(256)^{0.15} \cdot (256)^{0.1} = (256)^{0.25} = (2^8)^{1/4} = 2^2 = 4 \quad A$

⑨ $[(a^2 - b^2) \div (a + b)]^{-1} = \left[\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \cdot \frac{1}{(a + b)}\right]^{-1} =$

$\left[\frac{(b^2 - a^2)}{a^2 b^2} \cdot \frac{1}{(a + b)}\right]^{-1} = \left[\frac{-(a - b)(a + b)}{a^2 b^2} \cdot \frac{1}{(a + b)}\right]^{-1} =$

$\frac{-a^2 b^2}{a - b} \quad C$

⑩ $36e^x + 6e^{-x} = 35 \quad \text{Mult. by } e^x$

$36e^{2x} + 6 = 35e^x$

$36e^{2x} - 35e^x + 6 = 0$

$(9e^x - 2)(4e^x - 3) = 0$

$e^x = 2/9 \quad e^x = 3/4 \quad \left\{ \ln 2/9, \ln 3/4 \right\}$

sum of roots: $\ln 2/9 + \ln 3/4 = \ln\left(\frac{2}{9} \times \frac{3}{4}\right) =$

$\ln \frac{1}{6} = \ln 6^{-1} = -\ln 6 \quad D$

⑪ $2^a = 256 \quad 2^a = 2^8 \quad a = 8$
 $3^b = 27 \quad 3^b = 3^3 \quad b = 3$

$(2bx^3 - ay)^9 = (6x^3 - 8y)^9 =$ seven term polynomial with the sum of the coefficients equal to $(6 - 8)^9 = (-2)^9 = -512. \quad E$

⑫ $-2^{2^3} = -(2)^8 = -256 \quad A$

⑬ $\frac{(2n^{1/3} - 4n^{-2/3})}{2n^{-2/3}}, \frac{n^{2/3}}{n^{2/3}} = \frac{2n - 4}{2} = n - 2 \quad A$

⑭ $D \quad 2^{-(x+3)} + 1 = y$
 $\left(\frac{1}{2}\right)^{x+3} + 1 = y$

shifted 3 to left and up 1.

⑮ $(16)^{20} (125)^{27} = 2^{80} \cdot 5^{81} = (2 \cdot 5)^{80} \cdot 5 = 10^{80} \cdot 5$
 $10^{80} \cdot 5 \quad 10^1$ has 2 digits
 5×10^{80} has 81 digits D

⑯ $\left[\frac{2x^2 y^{-1}}{y^{-3}}\right]^4 = [2x^2 y^2]^4 = 16x^8 y^8 \quad E$

⑰ $\log_A B = \log_B A \quad \frac{\log B}{\log A} = \frac{\log A}{\log B}$

$[\log B]^2 = [\log A]^2 \quad a - \text{number squared}$
 $= a + \text{number squared}$

$[\log B]^2 = [\log \frac{1}{B}]^2 \quad A = \frac{1}{B} \quad \frac{1}{B} \cdot B = 1 \quad B$

⑱ $\left(\frac{1}{49}\right)^{2x-3} = 343^{3x-2}$

$(7^{-2})^{2x-3} = (7^3)^{3x-2} \quad -4x + 6 = 9x - 6$
 $12 = 13x$
 $\frac{12}{13} = x \quad C$

⑲ $3^6 = 729 \quad [\log_3 769] = 6$

$6^3 = 216 \quad 6^4 = 1296 \quad [\log_6 1024] = 3$

$5^5 = 3125 \quad 5^6 = 15625 \quad [\log_5 15124] = 5$

$[\log 1.9647 \times 10^{-5}] = -5$

$7^3 = 343 \quad 7^4 = 2401 \quad [\log_7 1299] = 3$

$6 + 3 + 5 - 5 + 3 = 12 \quad A$

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(20) $\frac{\log \frac{1}{25}}{\log 8} \cdot \frac{\log 4096}{\log 9} \cdot \frac{\log 36}{\log 625} \cdot \frac{\log 2}{\log 6} =$
 $\frac{-2 \log 5}{3 \log 2} \cdot \frac{12 \log 2}{2 \log 3} \cdot \frac{2 \log 6}{4 \log 5} \cdot \frac{\log 2}{\log 6} =$
 $\frac{-2 \log 2}{\log 3} = \frac{\log 2^{-2}}{\log 3} = \frac{\log \frac{1}{4}}{\log 3} = \log_3 0.25$ D

(21) $3^{\log x} = 3^x$ $\log_3 3^{\log x} = \log_3(3^x)$

$\log x \cdot \log 3 = \log 3 + \log x$

$\log x \cdot \log 3 - \log x = \log 3$

$\log x (\log 3 - 1) = \log 3$

$\log x = \frac{\log 3}{\log 3 - \log 10}$ mult by $\frac{1}{\log 3}$

$\frac{\log x}{\log 3} = \frac{1}{\log 3 - \log 10}$

$\log_3 x = \frac{\log 10}{\log 3/10} \left[\frac{\log 10}{\log 3/10} = \frac{\log_3 10}{\log_3 3/10} \right]$

$\log_3 x = \frac{\log_3 10}{\log_3 0.3}$

$3^{(\log_3 10) / (\log_3 0.3)} = x$ A

(22) $\frac{\log a}{\log 4} + \log_{64} b = c$

$\frac{3 \log a}{3 \log 4} + \log_{64} b = c$

$\log_{64} a^3 + \log_{64} b = c$

$\log_{64} a^3 b = c$ $a^3 b = 32$

$\log_{64} 32 = c$ $64^c = 32$ $2^{6c} = 2^5$ $c = \frac{5}{6}$ A

(23) $x^{4/3} - 6x^{2/3} + 8 = 0$

$(x^{2/3} - 4)(x^{2/3} - 2) = 0$

$x^{2/3} = 4$ $x^{2/3} = 2$

$x = \pm (4)^{3/2}$ $x = \pm (2)^{3/2}$

$\{-8, +8, \sqrt{2}, -2\sqrt{2}\}$
sum of roots = 0 E

(24) (a) $2^{x+2} - 2^x = 2^2 \cdot 2^x - 2^x = 2^x(4-1) = 3 \cdot 2^x$

(b) $2^{x+5} \div 2^x = 2^{x+5-x} = 2^5 = 32$

(c) $2^{14} = 16384$

(d) $2^{x+3} - 2^x = 2^3 \cdot 2^x - 2^x = 2^x(2^3-1) = 7 \cdot 2^x$

C is true

(25) $x^{(x+3)^2} = x^{64}$ -1, 0, 1 all give true statements

$(x+3)^2 = 64$

$x+3 = \pm 8$ $x = -3 \pm 8$ 5, -11

$| -1 | + | 0 | + | 1 | + | 5 | + | -11 | = 18$ D

(26) $x^3 - 2x^2 - 5x + 6 > 0$

$\begin{array}{r|rrrr} 1 & -2 & -5 & 6 \\ & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$ $(x^2 - x - 6)$
 $(x-3)(x+2)$

$(x-1)(x-3)(x+2) > 0$

$\{x: -2 < x < 1\} \cup \{x: x > 3\}$ A

(27) $3 + 1 + 1 + \frac{1}{4} + \frac{1}{3} + \frac{1}{16} + \frac{1}{9} + \dots$

two infinite series

$3 + 1 + \frac{1}{3} + \frac{1}{9} + \dots$ $\frac{3}{1-1/3} = 3(3/2) = 9/2$

$1 + \frac{1}{4} + \frac{1}{16} + \dots$ $\frac{1}{1-1/4} = 1 \cdot 4/3 = 4/3$

$\frac{9}{2} + \frac{4}{3} = \frac{27+8}{6} = \frac{35}{6}$ C

(28) $\log_4 a = \log_{15} b = \log_{25} (a+2b) = x$

$4^x = a$ $15^x = b$ $25^x = a+2b$ $\frac{b}{a} = \left(\frac{15}{4}\right)^x = \left(\frac{5}{3}\right)^x$

$4^x + 2 \cdot 15^x = 25^x$ Mult. by $\frac{1}{4^x}$

$1 + 2 \left(\frac{15}{4}\right)^x = \left(\frac{25}{4}\right)^x$ $1 + 2 \left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^{2x}$

$\left[\left(\frac{5}{3}\right)^x\right]^2 - 2 \left[\frac{5}{3}\right]^x - 1 = 0$ $\left(\frac{5}{3}\right)^x = \frac{2 \pm \sqrt{4-4(-1)}}{2}$

$\left(\frac{5}{3}\right)^x = 1 \pm \sqrt{2}$ $\frac{b}{a} > 0$ so $\frac{b}{a} = 1 + \sqrt{2}$ B

(29) $A = Pe^{rt}$ $A = 600,000 e^{10(1)}$ $= 600,000(e) \approx$

$600,000 \cdot 2.7 \approx 1.6$ million C

(30) $e^{\ln 6 - \ln 2} = e^{\ln 6/2} = e^{\log_e 3} = 3$

$e^{\ln 9 + \ln 1} = e^{\ln 9} = 9$

$e^{2 \ln 3 + \frac{1}{2} \ln 4} = e^{\ln 9 \cdot 2} = 18$

$e^{\ln 7 + \ln 3} = e^{\ln 21} = 21$

$\begin{vmatrix} 3 & 9 \\ 18 & 21 \end{vmatrix} =$

$3(21) - 9(18) =$

$63 - 162 = -99$

A