

Logs and Exponents Topic Test Answers

1. E
2. B
3. C
4. C
5. A
6. E
7. B
8. A
9. B
10. A
11. C
12. E
13. E
14. E
15. B
16. A
17. A
18. C
19. A
20. E
21. D
22. B
23. E
24. C
25. ~~B~~ A

Log and Exponential Logic Test Solutions

① $x-1 = \log_3 81$
 $3^{x-1} = 3^4$
 $x = 5$

Ans. E

② $\log_2(x+6) + \log_2(x-6) = 6$
 $x^2 - 36 = 2^6$
 $x^2 = 100$
 $x = 10$

Ans B

③ $\frac{2^{3x-2} \cdot 4^{x-2}}{32^{x-1}} =$
 $\frac{2^{3x-2} \cdot 2^{2x-4}}{2^{5x-5}} = \frac{1}{2}$

Ans C

④ $a = \log_4 7$
 $4^a = 7$
 $2^{2a} = 7$
 $2^a = \sqrt{7}$

Ans C

⑤ $(1+i)^{20} = [(1+i)^2]^{10}$
 $= (2i)^{10}$
 $= -1024$

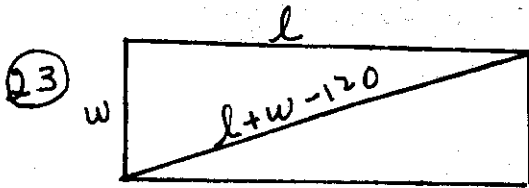
Ans A

⑥ $3^{x+4y} = \left(\frac{1}{9}\right)^{y-2x}$
 $3^{x+4y} = 3^{-2y+4x}$
 $x+4y = -2y+4x$
 $\frac{x}{y} = \frac{2}{1}$

Ans E

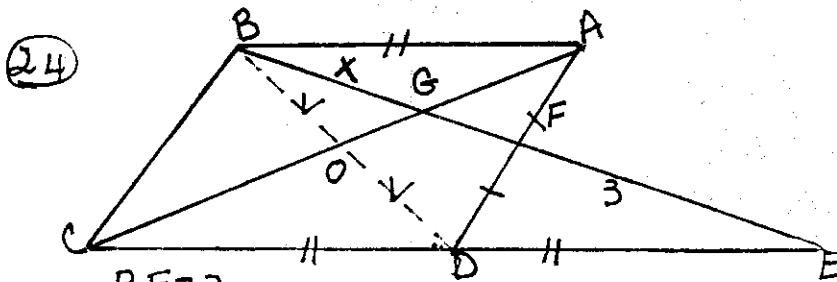
⑦ $10^a (20)^b = 4000$
 $10^a 20^b = 10^1 \cdot 20^2$
 $a+b = 1+2 = 3$

Ans B



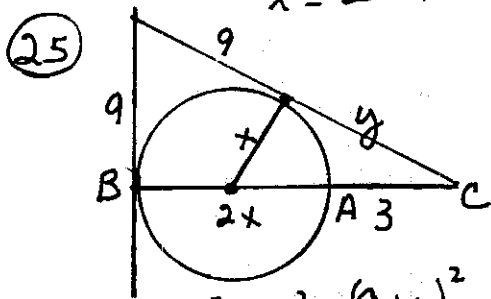
Ans E

$$\begin{aligned}
 lw &= 42000 \\
 l^2 + w^2 &= l^2 + 2lw + w^2 - 240l - 240w + 14400 \\
 l+w &= \frac{98400}{240} \\
 l+w &= 410 \\
 l+w-120 &= 410-120 \\
 &= 290
 \end{aligned}$$



Ans. A

$$\begin{aligned}
 BF &= 3 \\
 FE &= 3 \\
 BE &= 6 \\
 BF \text{ and } AO \text{ medians of } \triangle BDA \\
 \text{Med. intersect in a ratio of } 2:1. \\
 \text{Let } BG &= x \therefore GE = 2x \\
 x + 2x &= 6 \\
 3x &= 6 \\
 x &= 2
 \end{aligned}$$



$$\begin{aligned}
 (2x+3)^2 + 9^2 &= (9+y)^2 \\
 x^2 + y^2 &= (x+3)^2 \\
 4x^2 + 12x + 9 + 81 &= 81 + 18y + y^2 \\
 x^2 + y^2 &= x^2 + 6x + 9 \\
 4x^2 + 12x + 9 &= 18\sqrt{6x+9} + 6x + 9 \\
 2x^2 + 3x &= 9\sqrt{6x+9}
 \end{aligned}$$

$$\begin{aligned}
 4x^4 + 12x^3 + 9x^2 &= 486x + 729 \\
 4x^4 + 12x^3 + 9x^2 - 486x - 729 &= 0 \\
 \begin{array}{r|rrrrrr}
 4 & 4 & 12 & 9 & -486 & -729 \\
 & & 18 & 135 & 648 & 729 \\
 \hline
 & 4 & 30 & 144 & 162 & 0
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{9}{2} \\
 2x &= 9
 \end{aligned}$$

Ans. C

$$\textcircled{8} \quad b^a = a^b$$

$$\log_b b^a = \log_b a^b$$

$$a = b \log_b a$$

$$\log_b a = \frac{a}{b}$$

Ans A

$$\textcircled{9} \quad 7r+6 = r^3$$

$$r^3 - 7r + 6 = 0 \quad \text{2 pos and 1 neg root}$$

3 roots

$$\begin{array}{r|rrrr} z & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array} \quad \begin{array}{l} x \neq -3 \\ (x+3)(x-1) = 0 \\ x \neq 1 \end{array}$$

Ans B

$$\textcircled{10} \quad \log_b a = x$$

$$b^x = a$$

$$\log_a b^x = \log_a a$$

$$x \log_a b = 1$$

$$\frac{\log_a b}{\log_a a} = \frac{1}{x}$$

$$\frac{\log_a b}{\log_a a} = \frac{1}{x^2}$$

Ans A

$\textcircled{11}$

$$y = 2^{x-1} - 2$$

$$y_{int} = -\frac{3}{2}$$

$$x_{int} = 2$$

$$2 + (-\frac{3}{2}) = \frac{1}{2}$$

Ans C

$\textcircled{12}$

$$x^{2y} + x^{-2y} - 2 = 0$$

$$x^{4y} - 2x^{2y} + 1 = 0$$

$$(x^{2y} - 1)^2 = 0$$

$$x^{2y} = 1$$

$$\text{if } y = 0 \quad x \in \mathbb{R}$$

\therefore can't be determined

Ans E

13

$$\log_5 2 = a$$

$$\log_5 3 = b$$

$$\log_{25} 96 = \log_5 (2^5 \cdot 3^1)^{\frac{1}{2}}$$

Ans E

$$= \frac{1}{2} [5 \log_5 2 + \log_5 3]$$

$$= \frac{5a + b}{2}$$

14

$$\left(\frac{1}{a}\right)^{\frac{1}{b}} = x$$

$$(a^{-1})^{\frac{1}{b}} = x$$

Ans E

$$\frac{1}{a} = x^b$$

$$a = \frac{1}{x^b}$$

$$a^b = \left(\frac{1}{x^b}\right)^b$$

$$a^b = \left(\frac{1}{x}\right)^{b^2}$$

15

$$3^{20} > 32^x$$

Ans B

$$3^{20} > 2^{5x}$$

$$3^4 > 2^x$$

$$x \leq 6$$

16

$$\log_9 27 (243)^{\frac{1}{2}} = x$$

Ans A

$$3^{2x} = 3^3 \cdot 3^{\frac{5}{2}}$$

$$2x = \frac{11}{2}$$

$$x = \frac{11}{4}$$

$$(17) \quad 3^{2x-1} - 6 \cdot 3^x + 27 = 0$$

$$3^{2x} - 18 \cdot 3^x + 81 = 0$$

$$(3^x - 9)^2 = 0$$

$$3^x = 9$$

$$x = 2$$

Ans A

$$(18) \quad 27^x - 2(3)^{2x+2} + 4(3)^{x+3} - 216 = 0$$

$$3^{3x} - 18 \cdot 3^{2x} + 108 \cdot 3^x - 2 \cdot 3^3 = 0$$

$$(3^x - 6)^3 = 0 \quad \text{Ans C}$$

$$3^x = 6$$

$$x = \log_3 6$$

$$(19) \quad (1-2x)^{\frac{1}{3}}$$

$$4^{\text{th term}} \left(\frac{1}{3}\right) (1)^{-\frac{8}{3}} (-2x)^3$$

$$\frac{\frac{1}{3} \left(-\frac{2}{3}\right) \left(-\frac{5}{3}\right) (1)^{-\frac{8}{3}} (-2x)^3}{1 \cdot 2 \cdot 3}$$

$$\frac{10}{27} \cdot (-8)$$

$$\frac{10 \cdot 1}{27 \cdot 6} \cdot (-8) = \frac{-40}{81}$$

Ans A

$$(20) \quad \text{Let } a = \log_{40} 40\sqrt{3}$$

$$\therefore a = \log_{3^m} 45$$

$$4^a = 40\sqrt{3}$$

$$3^a = 45$$

$$\left(\frac{4}{3}\right)^a = \frac{8\sqrt{3}}{9}$$

$$\frac{2^{2a}}{3^a} = \frac{2^3}{3^{\frac{3}{2}}}$$

$$\therefore a = \frac{3}{2}$$

$$3^{\frac{3}{2}n} = 45$$

$$n^3 = 75$$

Ans E

$$(21) x = 5^{44}$$

$$\log x = 44 \log 5$$

$$\log x = 44 (\log 10 - \log 2)$$

$$\log x = 44 (1 - .201)$$

$$\log x = 30.756$$

no. of digits is 31

ans. D

$$(22) 2^a + 2^b = 3^c$$

2 to any power except 0 is even ans B

3 to any power is odd.

a or b must equal 0.

$$(23) \ln x^4 = (\ln x)^3$$

$$(\ln x)^3 - 4 \ln x = 0$$

ans E

$$\ln x (\ln x - 2) (\ln x + 2) = 0$$

$$\ln x = 0 \text{ or } \ln x = \pm 2$$

$$x = e^0$$

$$x = e^{\pm 2}$$

$$x = 1$$

$$x = e^2, e^{-2}$$

$$(24) 11 > a > b > c > 0$$

ans. C

$$\log a + \log b > \log c \rightarrow ab > c$$

$$\log b + \log c > \log a \rightarrow bc > a$$

$$\log a + \log c > \log b \rightarrow ac > b$$

Simply list them

$$\text{with } c=2 \quad 19 \Delta$$

$$c=3 \quad 21 \Delta$$

$$c=4 \quad 15$$

$$c=5 \quad 10$$

$$c=6 \quad 6$$

$$c=7 \quad 3$$

$$c=8 \quad 1$$

$$\frac{1}{75}$$

$$(25) \sum_{n=0}^{\infty} \frac{\cos nx}{2^n} \text{ where } \cos x = \frac{1}{5}$$

$\cos nx = \text{real part of } e^{inx}$ β

$\therefore \sum_{n=0}^{\infty} \left(\frac{e^{ix}}{2}\right)^n$ This is geometric.

$$\therefore S = \frac{1}{1 - \frac{e^{ix}}{2}} = \frac{2}{2 - e^{ix}} = \frac{2}{2 - \cos x - i \sin x}$$

$$= \frac{2(2 - \cos x + i \sin x)}{(2 - \cos x - i \sin x)(2 - \cos x + i \sin x)}$$

$$= \frac{2(2 - \cos x + i \sin x)}{4 - 4\cos x + \cos^2 x + \sin^2 x}$$

$$= \frac{2(2 - \cos x + i \sin x)}{5 - 4\cos x}$$

Take real part and evaluate

$$= \frac{2(2 - \frac{1}{5})}{5 - 4(\frac{1}{5})}$$

$$= \frac{18}{21}$$

$$= \frac{6}{7}$$