10

ROUND 1

1: _____

2: _____

3: _____

4: _____

5: _____

1. The arithmetic mean of x and 4x is 20. Compute x.

2. There are 120 red marbles and 80 blue marbles in a bag that contains 200 marbles. If only blue are to be added to the bag so that the probability of randomly drawing a blue marble from the bag becomes $\frac{2}{3}$, how many blue marbles must be added to the bag?

3. If $m + n = 3$ and $m^2 + n^2 = 6$, find the numberical value of $m^3 + n^3$.

4. Uncle Wilbur said to Aunt Emma, "Give me eight pieces of candy and we'll have equal amounts of candy. She replied, "No, you give me eight pieces of candy and then I will have twice as much as you." How many pieces of candy did Uncle Wilbur have originally?

5. Find a real number x such that $x = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$

ROUND 2

6: _____

7: _____

8: _____

9: _____

10: _____

6. Simplify: $\sqrt{\log_2 32 + \log_3 27 + \log_5 5}$

7. Two figures are such that the ratio of their surface areas is 1 : 4. If the volume of the smaller figure is 6 cubic units, find the volume, in cubic units, of the larger figure.

8. State the product of the roots of $16^{x^2+x+4} = 32^{x^2+2x}$

9. Solve the following for x and y. $\left[i = \sqrt{-1}\right]$

$$2 + 3ix - (4 + 5i) = (4i - 3)x - 5i$$

10. The first three terms of a sequence are in an arithmetic sequence with the first term being four more than the second term. All terms after the third are found by adding the two previous terms. If the fifth term is 340, find the ninth term.

ROUND 3

11: _____

12: _____

13: _____

14: _____

15: _____

11. What is the other endpoint of a line segment is one endpoint is $(2, 5)$ and the midpoint is $(3, 8)$?

12. How many terms are in the following sequence? 10, 17, 24, 31, . . . , 374

13. If $(ax + 7)(5x + b) = px^2 + 15x + 14$, find $a(b + p)$.

14. Solve for all real values of x: $4^x - 5 \cdot 2^x - 24 = 0$

15. A man has y friends. He is able to invite a different collection of two or more friends to his home every night for a year. What is the smallest possible value of y?

ROUND 4

16: _____

17: _____

18: _____

19: _____

20: _____

16. If $4(9a - 13b) = 6(a - 2b)$, what is the ratio of a to b? State this ratio in lowest terms.

17. There are 4 roads from Dog Town to Pequot and 5 roads from Pequot to North Haven. If Pete drives from Dog Town to North Haven and back, passing through Pequot in both directions, and he does not travel any road twice, how many different routes for the trip are possible.

18. Completely simplify the expression: $\sqrt[n]{\frac{4^{2n+1} + 2^{4n+1}}{6}}$

19. Find the exact value of $\left(\frac{AB}{C}\right)^5$, if $A = 6(\cos 15° + i \sin 15°)$, $B = 4(\cos 132° + i \sin 132°)$, and $C = 12(\cos 27° + i \sin 27°)$. [Remember that $i = \sqrt{-1}$].

20. Given quadrilateral ABCD with $m\angle D = 130°$ *and* $m\angle B = 72°$. The bisector of angle A intersects the line through C parallel to segment AB at poiint F. If quadrilateral ADCF is a trapezoid, find the measure, in degrees, of angle BCD.