

STATE MATH BOWL-Alpha

Hawaii 1993

Solution Time

Points

No Calculators

Within 1st minute

6

Within 2nd minute

3

Incorrect

0

Student Name: _____

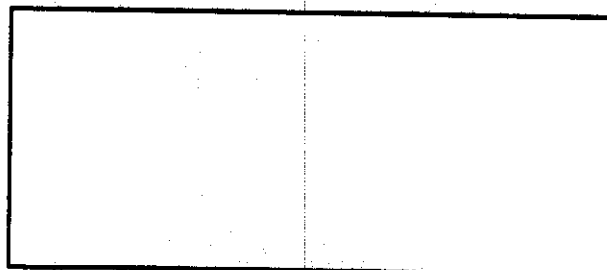
Team: _____

1

Question #1

If $a = 2(\cos 60^\circ + i \sin 60^\circ)$ and $b = 6(\cos 90^\circ + i \sin 90^\circ)$, find $\frac{a}{b}$ in simplified $x + yi$ form where x and y are real numbers.

Solution for #1



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2

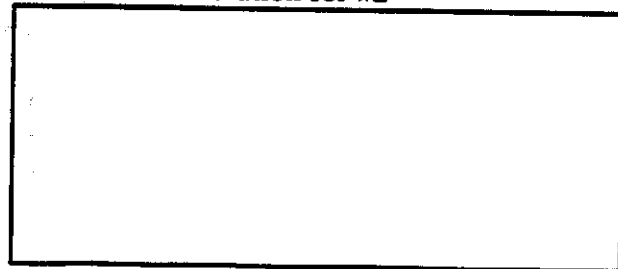
Question #2

If a certain real-valued function $f(x)$ with domain all real numbers satisfies the identity:

$$f(x+y) = f(x) + 2 \cdot f(y) \text{ for every pair of real numbers } (x,y)$$

What is $f(28.3)$?

Solution for #2



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12
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3

Question #3

Given the system:

$$x + y + z = 30$$

$$3x + 5y + 7z = 120$$

How many distinct ordered triples (x,y,z) exist such that x , y , and z are each positive integers?

Solution for #3

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No Calculators

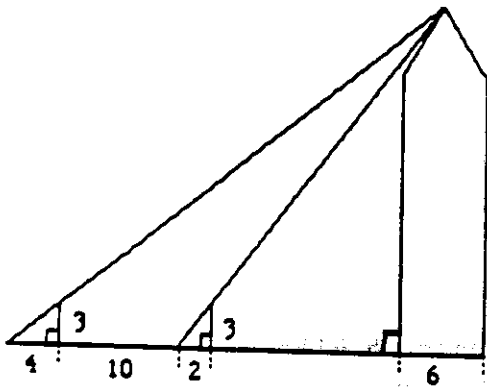
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Incorrect	0

4

Team: _____

Question #4

In order to estimate the height of an obelisk, an archaeologist lines up the end of shadow of the obelisk with that of a yardstick at two different times. Given that the archaeologist's measurements were as in the following diagram, estimate the height of the obelisk as closely as possible.



Solution for #4

5. Find the sum of the elements in:

$$\left\{ y \mid x^2 + \frac{4}{x^2} = 5 \text{ and } y + \frac{1}{y} = x \right\}$$

6. A square and equilateral triangle have equal areas. Find the ratio of the area of the circle inscribed in the square to the area of the circle inscribed in the equilateral triangle.
7. The zeroes of the polynomial $P(x) = x^3 - 8x^2 + 16x - 8$ are the length, width, and height of a rectangular solid. Find the total surface area of that rectangular solid.
8. Horky is $\frac{3}{5}$ of the way across a 2 mile trestle when he spots a train coming toward him. If the train's rate remains constant, Horky can just get off the trestle without being smacked by the train if he runs either way. How many miles is the train from the trestle?

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Incorrect

Points

6

3

0

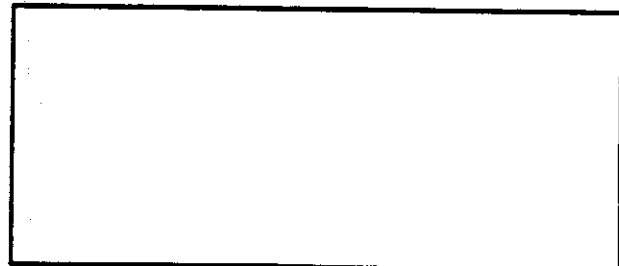
9

Question #9

What is the 1's digit of:

$$x = 2^{30} 3^{20} 7^{10} 11^{15} ?$$

Solution for #9



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3
0

10

Question #10 Find the value of x at which the function below has an inflection point:

$$f(x) = \int_0^x t^2 e^{-t^2} dt$$

Solution for #10

No Calculators

Within 1st minute

12

Within 2nd minute

6

Incorrect

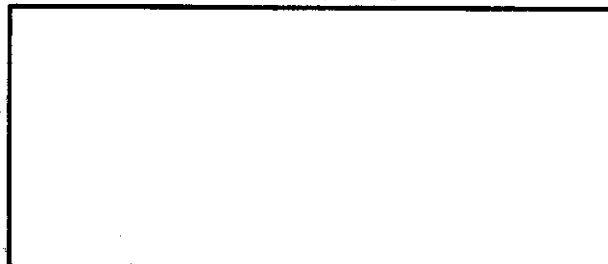
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Team: _____

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Question #11 If you know that among the students enrolled in Chemistry,
Calculus, or French:

- i) There are 18 students in at least two of the three classes.
 - ii) There are 33 students in the French and Chemistry classes combined.
 - iii) There are 24 people in exactly one of the three classes.
 - iv) There are 9 students taking both French and Chemistry.
 - v) There are 3 students taking all three classes.
- How many students are taking Calculus?

Solution for #11

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12

Team: _____

Question #12

In a class of exactly 10 students, exactly two have a first name of Jayne, exactly five have a middle name of Jayne, and exactly three have a last name of Jayne. If each student has exactly a first, middle, and last name, find the probability there is exactly one student in the class named Jayne Jayne.

Solution for #12

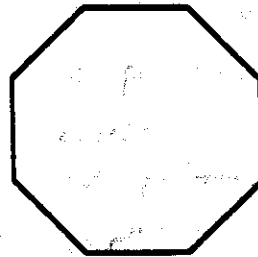
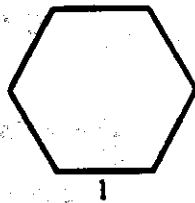
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13. Remember that $[x]$ stands for the greatest integer less than or equal to x and $\max\{a_1, a_2, \dots, a_n\}$ is the largest value of the members of the set.

Let p_n = the n^{th} prime, so that $p_1 = 2$, $p_2 = 3$, etc. and define $x_n = \max\{[x] \mid x^2 - p_{n+1}x + p_n = 0\}$.

Compute: $x_1 + x_2 + x_3$.

14. Rounded to the nearest tenth, find the positive difference between the areas of the 2 regular polygons shown.



15. Find the circumference of the intersection of:

$$x + y + z = 1$$

and

$$x^2 + y^2 + z^2 = 1$$

16. If a certain "unfair" coin will come up on the same side for 3 flips in a row with probability $1/2$, and it is more likely to come up heads than tails, what is the probability of getting a head with that coin?