Alpha State Team Ciphering Answers:

1. 91
2. $\sqrt{2}$
3. $\frac{1}{512}$
4. 600
5. 2
6. 8
7. -3
8. $\sqrt{3}$
9. 32
10. 2
11. 71
12. $\frac{3}{8}$
13. 37
14. 12

Alpha State Team Ciphering
Iowa State Solutions

① A: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47   15

B: $10800 = 100 \cdot 108 = 100 \cdot 12 \cdot 9$

$= 2^2 \cdot 5^2 \cdot 2^2 \cdot 3 \cdot 3^2 = 2^4 \cdot 3^3 \cdot 5^2$ 91

$n = 5 \cdot 4 \cdot 3$ 60

C: $(x-9)(x-1)^2(x+6) \le 0$

$- \; - \;\; | \;\; - \;\; + \;\; | \;\; + \; +$

$-6 \qquad\qquad 9$ 16

$-6 \le x \le 9$

$\overline{\quad 91 \quad}$

② $1991 = 11 \cdot 181$

$n = 1991$

$T = \frac{20}{10} = 2$

$\begin{cases} a + b = ab \\ a - b = 2 \end{cases}$ $\sqrt{2}$

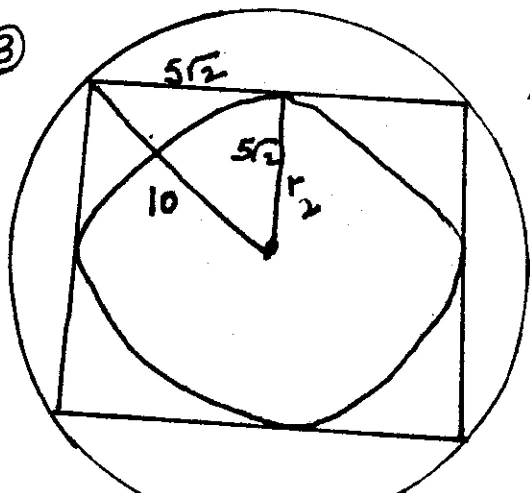
$b + 2 + b = (b+2)b$

$2b + 2 = b^2 + 2b$

$b^2 = 2$

$b = \sqrt{2}$

③



$A_1 = 100\pi$

$A_2 = 50\pi$ $\frac{1}{512}$

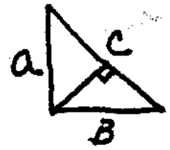
$r = \frac{1}{2}$

$A_{10} = ar^{n-1}$

$= 100\pi \left(\frac{1}{2}\right)^9$

$= \frac{100\pi}{512}$

$\frac{A_{10}}{A_1} = \frac{\frac{100\pi}{512}}{100\pi} = \frac{1}{512}$

④ 

$a + b + c = 120$

$h = 24$

"Guess" using triples

$3 + 4 + 5 = 12$

$30 + 40 + 50 = 120$

$A = \frac{1}{2} \cdot 30 \cdot 40$

$= 600$

verify $A = \frac{1}{2} \cdot \overset{12}{\cancel{24}} \cdot 50$

$= 600$

checks

600

⑤ $d = \sqrt{(x-2)^2 + (y+1)^2}$

$d^2 = x^2 - 4x + 4 + y^2 + 2y + 1$

$d^2 = \left(\frac{t}{t^2+1}\right)^2 - 4\left(\frac{t}{t^2+1}\right) + \left(\frac{1}{t^2+1}\right)^2 + \frac{2}{t^2+1} + 5$

2

$d^2 = 5 + \frac{3 - 4t}{t^2+1}$

$(d^2)' = \frac{4t^2 - 6t - 4}{(t^2+1)^2}$

$0 = 2t^2 - 3t - 2$

$0 = (2t+1)(t-2)$

$t = -\frac{1}{2}$ or $t = 2$

$- - \quad + \quad + \quad + \quad +$

$+ \quad + \quad +$

$-\frac{1}{2} \qquad 2$

$t = 2$ gives a min

$x = \frac{2}{5} \quad y = \frac{1}{5}$

$d = \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{6}{5}\right)^2} = 2$

⑥ $z = (-\sqrt{3}+i)^3$

$z = 0 + 8i$

$r_1 \quad -\sqrt{3}+i = 2 \text{ cis } 150^\circ$

$r_2 \quad = 2 \text{ cis } 270^\circ = 0 - 2i$

$r_3 \quad = 2 \text{ cis } 30^\circ = \sqrt{3} + i$

$a = 0$

$b = 8$

$c = 0$

$d = -2$

$e = \sqrt{3}$

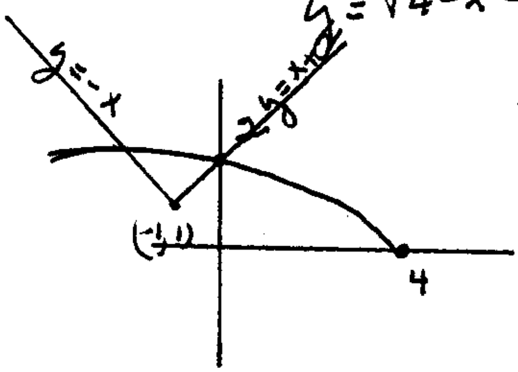
$f = 1$

$7 + 1\sqrt{3}$

$7 + 1 = 8$

8

⑦ Graph



$y = \sqrt{4-x} \rightarrow y^2 = 4 - x$

$y = |x+1| + 1$

when $x \geq -1$

$y = x + 2$

when $x \leq -1$

$y = -x$

$x^2 = 4 - x$

$x^2 + x + \frac{1}{4} = 4 + \frac{1}{4}$

$x + \frac{1}{2} = \pm\frac{\sqrt{17}}{2}$

$x = \frac{-\sqrt{17} - 1}{2}$

$\frac{-\sqrt{17}-1}{2} \leq x \leq 0$

$a = \frac{-\sqrt{17}-1}{2}$

$b = 0$

$[a] + [b] =$

$-3 + 0 = -3$

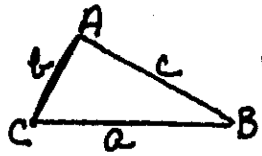
-3

⑧ $c^2 = \dfrac{a^3+b^3+c^3}{a+b+c}$

$ac^2 + bc^2 + \cancel{c^3} = a^3 + b^3 + \cancel{c^3}$

$c^2(a+b) = (a+b)(a^2-ab+b^2)$

$c^2 = a^2 - ab + b^2$



$c^2 = a^2 + b^2 - 2ab\cos c$

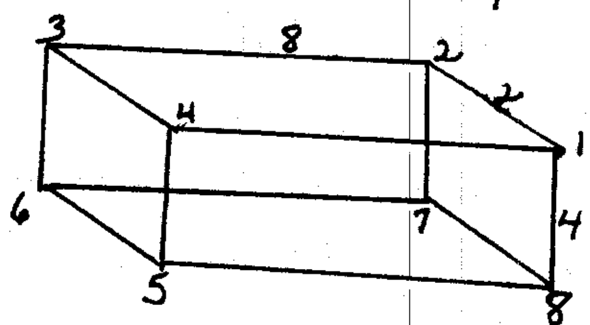
$\cancel{a^2} + \cancel{b^2} - 2ab\cos c = \cancel{a^2} - ab + \cancel{b^2}$

$\cos c = \dfrac{-ab}{-2ab} = \dfrac{1}{2}$

$\therefore \tan c = \dfrac{\sqrt{3}}{1}$

$\dfrac{\sqrt{3}}{1}$

⑨



$d_1 = \quad = 4$

$d_2 = \sqrt{2^2+4^2} = 2\sqrt{5}$

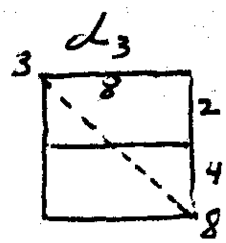
$d_3 = \sqrt{8^2+6^2} = 10$

$d_4 = \sqrt{8^2+4^2} = 4\sqrt{5}$

$d_5 = \quad = 8$

$d_6 = \sqrt{8^2+2^2} = 2\sqrt{17}$

$d_7 = \quad = 2$



32

$S = 24 + 6\sqrt{5} + 2\sqrt{17}$

$a + b + c = 32$

(10) $$\sum_{n=1}^{\infty} \frac{2n}{(n+1)!} =$$

2

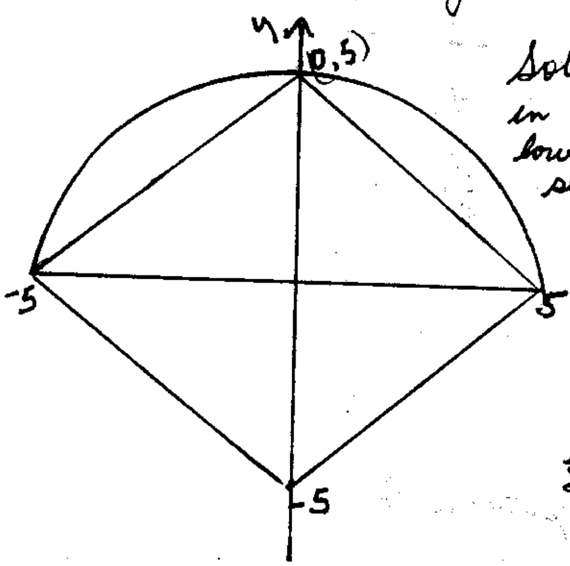
$$\sum_{n=1}^{\infty} \frac{2n+2-2}{(n+1)!} =$$

$$\sum \left(\frac{2(n+1)}{(n+1)!} - \frac{2}{(n+1)!}\right) =$$

$$2 \sum \left(\frac{1}{n!} - \frac{1}{(n+1)!}\right) =$$

$$2\,[e-(e-1)] = 2$$

(11) Graph: $(0 \le y \le \sqrt{25-x^2} \cup |x|+|y| \le 25$



Sol set consists of lattice pts in semi circle + lattice pts in lower half of rhombus.

71 pts

semicircle)

when $y = 0$	11 pts
$y = 1$	9 pts
$y = 2$	9 pts
$y = 3$	9 pts
$y = 4$	7 pts
$y = 5$	1 pt
	46 pts
$\frac{1}{2}$ rhombus	
$y = -1$	9 pts
$y = -2$	7 pts
$y = -3$	5 pts
$y = -4$	3
$y = -5$	1
	25 pts

total $= 46 + 25$

$= 71$ pts

(12) P, Q and R can be T or F

We want

$$P(P \cup Q)_F \text{ and } P(R)_F$$

$$P(P \cup Q)_F = P(P)_F + P(Q)_F - P(PQ)_F$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2}$$

$$= \frac{3}{4}$$

$$P(R)_F = \frac{1}{2}$$

$$\therefore P(P \cup Q)_F \wedge P(R)_F = \frac{3}{4}\cdot\frac{1}{2}$$

$$= \frac{3}{8}$$

$\frac{3}{8}$

(13)

7	6	5	4	3	2	1

7! ways to assign persons to days with no 2 having same birthday

7	7	7	7	7	7	7

7^7 ways in all to assign persons

$$\therefore P(B) = \frac{7!}{7^7} = \frac{6!}{7^6}$$

$$= \frac{720}{117649}$$

sum of digits is 37

37

(14) $\sum_{k=0}^{\infty} \frac{2k+1}{2^{k-1}} =$

$\sum_{k=0}^{\infty} \frac{2(2k+1)}{2^{k}} =$ $\underline{\underline{12}}$

$4\sum_{k=0}^{\infty} \frac{k}{2^{k}} + 2\sum_{k=0}^{\infty} \frac{1}{2^{k}} =$

$\sum_{k=0}^{\infty} \frac{k}{2^{k}} = 0 + \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots$ $\sum_{k=0}^{\infty} \frac{1}{2^{k}} = 1 + \frac{1}{2} + \frac{1}{4} + \dots$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \quad S_n = 1$ $S_n = \frac{1}{1-\frac{1}{2}} = 2$

$\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots \quad S_n = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{2}$

$\frac{1}{8} + \frac{1}{16} + \dots \quad S_n = \frac{\frac{1}{8}}{1-\frac{1}{2}} = \frac{1}{4}$

Total sum $= \frac{1}{1-\frac{1}{2}} = 2$

$4(2) + 2(2) =$

$8 + 4 = 12$