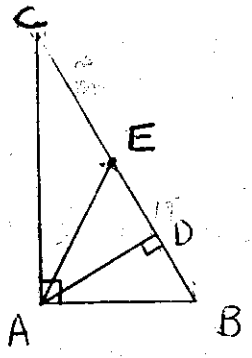
ALPHA DIVISION STATE TEAM: QUESTION 1



Given: right triangle ABC with altitude $\overline{AD}$ and median AE.

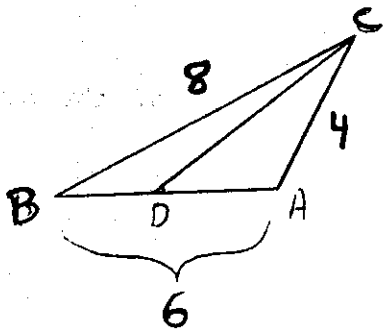
AC = 4; AB = 3.

Let w = length of $\overline{AD}$. Let y = length of $\overline{CD}$.

Let x = length of $\overline{AE}$. Let z = length of $\overline{DE}$.

Find the value of $\frac{x(w+y)}{z}$

ALPHA DIVISION STATE TEAM: QUESTION 2



In triangle ABC, CD bisects angle C.

Find the area of triangle ACD.

ALPHA DIVISION STATE TEAM: QUESTION 3

Solve each equation and find the value of ac - bd.

1. $\log(16a + 4) = 2$

2. $\log_4 b + \log_4(b+12) = \log_2 8$

3. $(\log_{81} 4)(\log_2 c) = \log_9 100$

4. $(\log_5 d)^2 - \log_5 d^6 + \log_2 512 = 0$

ALPHA DIVISION STATE TEAM: QUESTION 4

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The expression $BB^{-1} + AB$ may be expressed as a 3X3 matrix. Find the sum of the nine elements of that matrix.

QUESTION 5: ALPHA STATE TEAM

Let a = the term of $\left(x^4 + \frac{1}{x^2}\right)^6$ which has no x.

Let b = the coefficient of the third term of $(x - 2y)^5$.

Let c = the sum of the elements in the eighth row of Pascal's triangle.

Let d = the sum of the coefficients of the expansion $(2x + y)^4$.

Find the value of (b + d) - (a + c).

ALPHA DIVISION STATE TEAM: QUESTION 6

Let A= Units digit of 1987^{1989}

Let B= LCM of 1,2,3,4,5,6,7,8,9,10

Let C= number of digits in the product $(16^{10})(25^{19})$

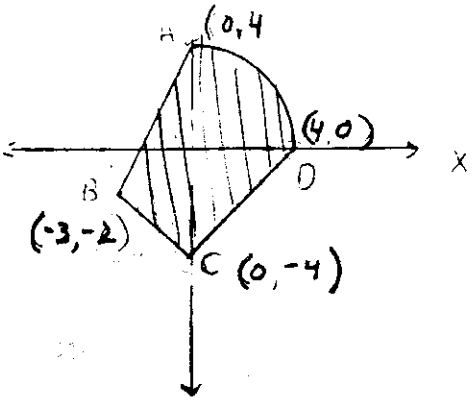
Find the value of A+B+C

ALPHA DIVISION STATE TEAM: QUESTION 7

Solve for x: $(4x+7)^2(x+1)^3(2x-1)^4(x-2)^6(4-x)^3 \geq 0$.

ALPHA DIVISION STATE TEAM: QUESTION 8

Find the area of the shaded region if AD is an arc of a circle with center at the origin.



ALPHA DIVISION STATE TEAM: QUESTION 9

With only the cold water faucet turned on a tub can be filled in 20 minutes. The hot water faucet alone can fill the tub in 25 minutes. How many minutes does it take to fill the tub if both faucets are turned on?

ALPHA DIVISION STATE TEAM: QUESTION 10

Evaluate $\sum_{n=1}^{\infty} \frac{1}{n^2+5n+6}$

ALPHA DIVISION STATE TEAM: QUESTION 11

A hexagon has nine diagonals. Find the sum of the lengths of the 9 diagonals of a regular hexagon whose side is 2.

ALPHA DIVISION STATE TEAM: QUESTION 12

An equilateral triangle is inscribed in the parabola whose equation is $2y = x^2$. If one vertex of the triangle is taken at the origin, find the area of the triangle.