

Mu Alpha Theta National Convention 2004

Alpha Sequences & Series Test

For all questions, answer E, "NOTA" means none of the above answers is correct.

1. How many of the following are true statements?

I.  $\sum_{k=0}^5 a^k + \sum_{n=6}^{10} a^n = \sum_{b=0}^{10} a^b$

II.  $\sum_{n=1}^{10} (5+n) = \sum_{m=0}^9 (4+m)$

III.  $2\sum_{k=3}^7 k^2 = \sum_{k=3}^7 2k^2$

IV.  $\sum_{r=2}^8 (2r-3) = \sum_{s=3}^9 (2s-5)$

- a) 1      b) 2      c) 3      d) 4      e) NOTA

2. Given an equilateral has a side of length 8. Line segments join the midpoints of the sides to form a second triangle. Then, line segments join the midpoints of the sides of the second triangle to form a third triangle. If this process continues, what is the limiting sum of the perimeters of the triangles so formed?

- a) 24      b) 48      c) 96      d) 108      e) NOTA

3. A computer is programmed to take as input a positive integer and print out its common English name, and then counts the number of characters in that name (excluding spaces and hyphens). For example, given 21, it prints "twenty-one", then "9". Someone accidentally loops the program, so it receives as input the number it just put out. Eventually, the computer will print out the same number over and over. What is that number?

- a) 1      b) 3      c) 4      d) 6      e) NOTA

4. Find the sum of the following series:  $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{25}+\sqrt{24}}$

- a) 1      b) 2      c) 3      d) 4      e) NOTA

5. The arithmetic mean of 2 positive numbers exceeds its geometric mean by 50. By how much does the square root of the larger of the 2 numbers exceed the square root of the smaller?

- a) 10      b) 20      c) 40      d) 50      e) NOTA

6. All of the circles in an infinite chain of successively tangent circles of decreasing size are tangent to both sides of a  $60^\circ$  angle. If the length of the radius of the largest of the circles in the chain is 105, find the sum of the lengths of the radii of all the circles in the chain.

- a) 157.5      b) 210      c) 275.5      d) 315      e) NOTA

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7. The  $n$ th term of a sequence is signified by  $t_n$ . If  $t_1 = 1$ ,  $t_2 = 3$ ,  $t_3 = 5$ , and if  $t_n = t_{n-1} + t_{n-2} - t_{n-3}$  for all  $n > 3$ , find the sum of the first 30 terms of the sequence.
- a) 200      b) 400      c) 600      d) 800      e) NOTA
8. Consider all positive integers between 3000 and 4000 inclusive which, when divided by any of the first 3 primes, leaves a remainder of 1. Find the sum of all these integers.
- a) 82,302      b) 100,428      c) 118,864      d) 196,400      e) NOTA
9. The real roots of  $2x^3 - 7x^2 + kx + 2 = 0$  are in geometric progression. Find the difference of these 3 roots, if you subtract them from the greatest to least.
- a) 2      b) 1.5      c) 1      d) 0.5      e) NOTA
10. Find the sum of the infinite series whose  $n$ th term is  $\frac{7^{n-1}}{10^n}$ .
- a)  $\frac{1}{4}$       b)  $\frac{1}{3}$       c)  $\frac{1}{2}$       d)  $\frac{3}{4}$       e) NOTA
11. The  $n$ th term of a sequence is signified by  $t_n$ . For all  $n > 1$ ,  $t_1 + t_2 + t_3 + \dots + t_n = n^2 t_n$  and  $t_1 = \frac{1}{2}$ . Find the exact value of  $t_1 + t_2 + t_3 + \dots + t_{50}$ .
- a)  $\frac{49}{50}$       b)  $\frac{50}{51}$       c)  $\frac{51}{52}$       d)  $\frac{52}{53}$       e) NOTA
12. An amoeba propagates by simple division and each split takes 3 minutes to complete. When such an amoeba is put into a glass container with a nutrient fluid, it is full of amoebas in one hour. How long would it take the container to be full of amoebas if we started with 2 amoebas (in minutes)?
- a) 30      b) 42      c) 50      d) 57      e) NOTA
13. Evaluate:  $\sum_{k=1}^{n+1} (2k-1) - \sum_{k=1}^n (2k-1)$
- a)  $2n$       b)  $2n + 1$       c)  $2n - 1$       d)  $n$       e) NOTA

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14. A person wins \$10 million in the Florida lottery, and the amount is paid in yearly installments of half a million dollars each year for 20 years. What is the present value of his winnings (to the nearest dollar) if you assume he earns 10% interest, compounded annually?

- a) \$4,110,662    b) \$4,256,782    c) \$4,301,244  
d) \$4,507,298    e) NOTA

15. Evaluate:  $\sum_{i=1}^{\infty} \frac{i}{5^i}$

- a)  $\frac{1}{4}$     b)  $\frac{5}{16}$     c)  $\frac{1}{2}$     d)  $\frac{3}{4}$     e) NOTA

16. Four consecutive terms of a series are 2, 5, 10, and 17. If the series is continued, find the square root of the sum of the arithmetic and geometric means between the 1<sup>st</sup> and 7<sup>th</sup> terms.

- a) 5    b)  $5\sqrt{5}$     c) 6    d)  $6\sqrt{6}$     e) NOTA

17. What is the product of the first 10 terms of a geometric series whose 1<sup>st</sup> term is 1 and whose 10<sup>th</sup> term is 2?

- a) 15    b) 16    c) 30    d) 32    e) NOTA

18. In an arithmetic progression, the 25<sup>th</sup> term is 2552, and the 52<sup>nd</sup> term is 5279. What is the 79<sup>th</sup> term?

- a) 7725    b) 7982    c) 8006    d) 8221    e) NOTA

19. The 4<sup>th</sup> term of a sequence is 4 and the 6<sup>th</sup> term is 6. Every term after the 2<sup>nd</sup> is the sum of the 2 preceding terms. What is the 8<sup>th</sup> term of this sequence?

- a) 8    b) 10    c) 12    d) 14    e) NOTA

20. What is the numerical coefficient of  $x^{27}$  in the complete expansion of  $(x^2 + 5x + 2)^5(x^2 - 7x + 3)^9$ ?

- a) -38    b) 38    c) -45    d) 45    e) NOTA

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21. If a boy deposits 1 cent on Sept. 1, 2 cents on Sept. 2, 4 cents on Sept. 3, 8 cents on Sept. 4, and so on, how much would he deposit on Sept. 30?
- a) \$1,342,177.28      b) \$2,684,354.56      c) \$5,368,709.12  
d) \$10,737,418.24      e) NOTA
22. Insert 4 geometric means between  $\frac{81}{2}$  and  $\frac{-16}{3}$ .
- a) 27, 18, 12, 8      b) 27, -18, 12, -8      c) -27, 18, -12, 8  
d) -27, -18, -12, -8      e) NOTA
23. Find the values of  $x$  for which the following geometric series may be summed:  
 $5 + 5(x-3) + 5(x-3)^2 + \dots$
- a)  $2 \leq x \leq 4$       b)  $2 < x \leq 4$       c)  $2 \leq x < 4$       d)  $2 < x < 4$       e) NOTA
24. To create a sinking fund to provide capital to purchase some new equipment, a company deposits \$25,000 into an account on January 1 every year for 10 years. If the account earns 12% interest compounded annually, how much is in the sinking fund immediately after the 10<sup>th</sup> deposit is made (to the nearest thousand dollars)?
- a) \$312,000      b) \$439,000      c) \$525,000      d) \$624,000      e) NOTA
25. Find the sum of the 1<sup>st</sup> 22 terms of the geometric progression having a 1<sup>st</sup> term of  $i$ , and ratio of  $(1+i)$ , where  $i = \sqrt{-1}$ . Answer in the form  $c + di$ , where  $c$  and  $d$  are real numbers.
- a)  $1 + 2048i$       b)  $1 - 2048i$       c)  $-1 + 2048i$       d)  $-1 - 2048i$       e) NOTA
26. In an arithmetic progression, the  $n$ th term is denoted by  $t_n$ . If  $3t_3 = 6t_6$ , find the value of  $9t_9$ .
- a) 1      b) 3      c) 6      d) 9      e) NOTA
27. If  $x_n^2 - x_{n-1}x_{n+1} = (-2)^n$  for  $n \geq 1$ , and  $x_0 = x_1 = 1$ , then  $x_3 = ?$
- a) 1      b) -3      c) 3      d) 5      e) NOTA

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28. Find the sum of the first 100 terms of the sequence:  $\frac{2}{1 \cdot 3}, \frac{2}{3 \cdot 5}, \frac{2}{5 \cdot 7}, \dots, \frac{2}{(2n-1)(2n+1)}$

- a)  $\frac{198}{199}$       b)  $\frac{199}{200}$       c)  $\frac{200}{201}$       d)  $\frac{201}{202}$       e) NOTA

29. I) Given an arithmetic sequence for which only the 1<sup>st</sup> and 2<sup>nd</sup> terms are known, then it is possible to find the nth term.

II) If the only known information about a finite arithmetic sequence is its 1<sup>st</sup> term and its last term, then it is possible to find the sum of the sequence.

If statement I is true, write the digit 1, if it is false, write the digit 2. If statement II is true, write the digit 3, if it is false, write the digit 4. Write your answer as a 2-digit number, with the 1<sup>st</sup> digit you get from statement I and the 2<sup>nd</sup> digit you get from statement II.

- a) 13      b) 24      c) 23      d) 14      e) NOTA

30. Find the exact numerical value of  $1 + 3\left(\frac{1}{3}\right) + 5\left(\frac{1}{3}\right)^2 + 7\left(\frac{1}{3}\right)^3 + 9\left(\frac{1}{3}\right)^4 + \dots$

- a) e      b) 3      c)  $\pi$       d) 5      e) NOTA

T.B. 1 A digital clock displays "1:23" and one notices that the three digits form an arithmetic sequence. How many times within the next hour are digits displayed in an arithmetic sequence?

T.B. 2 A harmonic progression is a sequence of numbers such that their reciprocals are in arithmetic progression. Let  $S_n$  represent the sum of the first n terms of the harmonic progression; for example,  $S_3$  represents the sum of the first three terms. If the first three terms of a harmonic progression are 3, 4, and 6, then  $S_4 = ?$ .