Mu Alpha Theta National Convention: Seattle, 1997

Alpha Sequences and Series Topic Test

1. The sum of the first 17 terms of an arithmetic sequence with common difference 6 is 442. What is the 12th term of the sequence?

   (A) 38 (B) 44 (C) 46 (D) 48 (E) NOTA

2. The sum of the first six terms of a geometric series is 399, and the common ratio is $\frac{5}{2}$. What is the third term?

   (A) $\frac{185}{13}$ (B) $\frac{200}{13}$ (C) $\frac{212}{13}$ (D) $\frac{223}{13}$ (E) NOTA

3. Determine the sum of the 21st and 22nd terms of the sequence that begins
   12, 11, 10,10, 8, 9, 6, 8, ...

   (A) 6 (B) 1 (C) -1 (D) -6 (E) NOTA

4. A sequence of four numbers exists such that the first three numbers form an arithmetic sequence of common difference 8 and the last three numbers form a geometric sequence of common ratio $\frac{4}{3}$. What is the fourth number in the original sequence?

   (A) $\frac{125}{3}$ (B) 42 (C) $\frac{127}{3}$ (D) $\frac{128}{3}$ (E) NOTA

5. Evaluate: $\sum_{n=1}^{22} 2n^3 - n^2 + 3$

   (A) 102,543 (B) 107,296 (C) 112,343 (D) 124,289 (E) NOTA

6. Evaluate: $\sum_{n=3}^{18} 3n^2 + 2n - 23$

   (A) 6280 (B) 6472 (C) 6691 (D) 6733 (E) NOTA

7. Evaluate: $\sum_{n=17}^{76} \frac{1}{n^2 - 9n + 20}$

   (A) $\frac{23}{360}$ (B) $\frac{1}{15}$ (C) $\frac{5}{72}$ (D) $\frac{13}{180}$ (E) NOTA

8. The first, fourth, and sixth terms of an arithmetic sequence with common difference 6 form a geometric sequence. What is the seventh term of the arithmetic sequence?

(A) -18 (B) -11 (C) -2 (D) 11 (E) NOTA

9. Which of the following is equal to $\sum_{n=21}^{250} \log\left(\frac{n+3}{n+1}\right)$?

(A) log 75 (B) $3\log 5$ (C) log 126 (D) log 252 (E) NOTA

10. A recursively defined sequence has first term 17 and $f_n = -2f_{n-1} + 48$. What is the sixth term in this sequence?

(A) -25 (B) -16 (C) -4 (D) 8 (E) NOTA

11. Find the limit of the sequence: $\frac{16}{7}, \frac{18}{10}, \frac{20}{13}, \ldots$

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) 1 (D) $\frac{3}{2}$ (E) NOTA

12. Evaluate: $\sum_{n=111}^{143} (-1)^n n^2$

(A) -22045 (B) -21998 (C) -16401 (D) -13452 (E) NOTA

13. A sequence of numbers begins 4,7 and each successive term is the previous term minus the term before that. What is the sum of the first 1000 terms?

(A) 11 (B) 14 (C) 10 (D) 3 (E) NOTA

14. A blob in a petri dish is originally one μm (10^{-6} m) in diameter, and doubles its diameter every twenty seconds. What is the diameter of the blob, to the nearest meter, after 8 minutes?

(A) 17 (B) 22 (C) 36 (D) 41 (E) NOTA

15. A rubber ball dropped thirty feet bounces $\frac{1}{3}$ of the height from which it fell on each bounce. How far will it travel before coming to rest?

(A) 36 (B) 72 (C) 48 (D) 60 (E) NOTA

16. Let S be the nth term of the arithmetic series 9, 14, 19,..., and T be the nth term of the series 156, 158, 160,... For what value of n will S=T?

(A) 42 (B) 50 (C) 68 (D) 76 (E) NOTA

17. What is the coefficient of the x^4 term in the expansion of $(3x-2)^{12}$?

(A) 10,264,320 (B) 8,252,224 (C) 15,361,488 (D) 7,723,816 (E) NOTA

18. The sum of the squares of the first and fourth terms of an arithmetic series is 1205, while the sum of the squares of the second and third terms is 881. What is the product of these four terms?

(A) 95,200 (B) 132,860 (C) 112,385 (D) 75,384 (E) NOTA

19. The numbers one to 10,000 are written in a row. Joe starts at 4 and circles every 88th number, while Jim starts at 14 and circles every 144th number. What is the smallest possible non-zero separation between one of Jim's numbers and one of Joe's numbers?

(A) 8 (B) 4 (C) 2 (D) 1 (E) NOTA

20. Find the sum of all the multiples of 27 between 126 and 8543, inclusive.

(A) 438,228 (B) 640,812 (C) 965,342 (D) 1,352,052 (E) NOTA

21. Two positive real geometric means are inserted between 7 and 448. What is the larger of the two?

(A) 28 (B) 56 (C) 112 (D) 140 (E) NOTA

22. Evaluate: $\sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1}$

(A) $\frac{8}{11}$ (B) $\frac{3}{4}$ (C) $\frac{4}{5}$ (D) $\frac{17}{22}$ (E) NOTA

23. Evaluate: $\sum_{n=45}^{123} 12n - 345$

(A) 45,028 (B) 52,377 (C) 59,179 (D) 64,242 (E) NOTA

24. What is the 82nd term of an arithmetic series that begins with -17 and has common difference 6?

(A) 457 (B) 463 (C) 469 (D) 475 (E) NOTA

25. Find the sum of the series: $3+6+12+24+\ldots+3\times 2^{20}$

(A) $3\left(2^{21}-1\right)$ (B) $3\left(2^{21}+1\right)$ (C) $3\left(2^{20}-1\right)$ (D) $3\left(2^{20}+1\right)$ (E) NOTA

26. Find the sum of the series: $48+36+27+\cdots$

(A) 215 (B) 192 (C) 144 (D) 96 (E) NOTA

27. Find the sum of the series: $30-25+\frac{125}{6}-\cdots$

(A) $\frac{101}{6}$ (B) $\frac{180}{11}$ (C) $\frac{35}{2}$ (D) $\frac{88}{5}$ (E) NOTA

28. Find the sum of the series: $1+4+\cdots+(n-1)^2+n^2$

(A) $\frac{n(n+1)(2n+1)}{6}$ (B) $\frac{n(n+1)^2}{4}$ (C) $\frac{n^2(2n+1)}{2}$
(D) $\frac{2(n+1)^2(2n-1)}{9}$ (E) NOTA

29. Find the sum of the first 30 terms of the Fibonacci Sequence.

(A) 176,788 (B) 180,778 (C) 194,218 (D) 217,838 (E) NOTA

30. What is the coefficient of the x^3 term in the Taylor Series of e^x?

(A) $\frac{1}{27}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{6}$ (E) NOTA