Mu Alpha Theta National Convention 2004
Alpha Individual
Answers

#	Answer	#	Answer
1	D	18	A
2	B	19	B
3	A	20	B
4	C	21	D
5	C	22	D
6	A	23	D
7	D	24	A
8	B	25	B
9	D	26	D
10	B	27	D
11	A	28	B
12	A	29	B
13	D	30	C
14	D	TB1	4
15	C	TB2	$\frac{2}{n}$
16	B	TB3	
17	A		

1. D

$$\left(0-64i\right)=64\left(\cos 270^\circ + i\sin 270^\circ\right)$$

$$\left(0-64i\right)^{\frac{1}{3}} = 64^{\frac{1}{3}}\left(\cos\frac{270^\circ+360k^\circ}{3}+\sin\frac{270^\circ+360k^\circ}{3}\right)$$

$$if\ \ k=0 \qquad 4\left(\cos 90^\circ + i\sin 90^\circ\right)=4i$$

$$if\ \ k=1 \qquad 4\left(\cos 210^\circ + i\sin 210^\circ\right)=4\left(\frac{-\sqrt{3}}{2}+i\left(\frac{-1}{2}\right)\right)=-2\sqrt{3}-2i$$

$$if\ \ k=2 \qquad 4\left(\cos 330^\circ + i\sin 330^\circ\right)=4\left(\frac{\sqrt{3}}{2}+i\left(\frac{-1}{2}\right)\right)=2\sqrt{3}-2i$$

2. B $\log_b\left(\frac{b}{3}\right)=\log_b b-\log_b 3=1-\log_b\frac{18}{6}=1-\left(\log_b 18-\log_b 6\right)=1-\left(a-c\right)=1-a+c$

3. A $tan\frac{3\pi}{4}=-1,\ \ \text{Arccsc}\left(-1\right)=-\frac{\pi}{2},\quad \cos\frac{-\pi}{2}=0,\ \ \text{Arcsin}\left(0\right)=0$

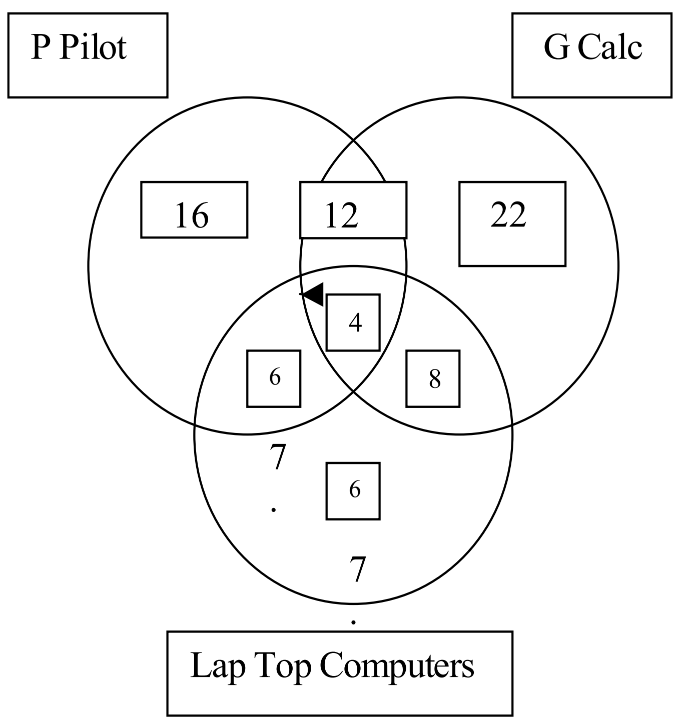
4. C $g\left(f\left(x\right)\right)=\frac{1}{\sqrt{1-x^2}},\quad 1-x^2>0,\quad -1<x<1\ \text{ or }\ \left(-1,1\right)$

5. C Using Descartes' Rule of Sign

1. $-\quad -\quad -$ no negative roots
2. $+\quad +\quad +$ no negative roots
3. $-\quad -\quad -\quad +$ one negative root

6. A Using a Venn Diagram

$16+6+22+6+8+12+4=7$



7. D $\frac{1}{e^{2\ln x}}=\frac{1}{e^{\ln x^2}}=\frac{1}{x^2}$

8. B

1. $\frac{1\text{-}\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x$ false

2. $\frac{\cos x \cos x}{\sin x} + \frac{\sin^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$ true

3. $\frac{\sec^2 x - 1}{\sec x + 1} + \frac{\sec x + 1}{\sec x + 1} = \frac{\sec x(\sec x + 1)}{\sec x + 1} = \sec x$ true

9. D. The period of $4\sin 10\pi t$ is $\frac{2\pi}{10\pi} = 0.2$ and the period of $10\cos 4\pi t$ is $\frac{2\pi}{4\pi} = 0.5$.
The least common multiple of 0.2 and 0.5 is 1.

10. B $\begin{vmatrix} \overset{\vee}{i} & \overset{\vee}{j} & \overset{\vee}{k} \\ -9 & 4 & -8 \\ 6 & -2 & 4 \end{vmatrix} = \overset{\vee}{i}(16-16) - \overset{\vee}{j}(-36+48) + \overset{\vee}{k}(18-24) = 0\overset{\vee}{i} - 12\overset{\vee}{j} - 6\overset{\vee}{k}$

11. A $e = 2$ $\therefore$ hyperbola, $p = 4$ distance from focus to directrix since directrix is horizontal and below the polar axis $r = \frac{ep}{1 - e\sin\theta} = \frac{8}{1 - 2\sin\theta}$

12. A Since circular number $= \frac{n!}{n}$ and since reflective number $= \frac{n!}{2n} = \frac{6!}{2(6)} = 60$

13. D. $a^{-2}(a^{x-5}) = a^{-1(x+2)}(a^{15-3x}) \rightarrow x - 7 = -x - 2 + 15 - 3x \rightarrow x - 7 = -4x + 13 \rightarrow x=4$

14. D.
$m = \frac{1-3}{2+2} = \frac{-2}{4} = \frac{-1}{2}$ $y - 1 = \frac{-1}{2}(x-2)$ $2y - 2 = -x + 2$ $2y - 2 = -(t+1) + 2$ $y = \frac{-1}{2}t + \frac{3}{2}$

15. C. $f(x) = 1 - f(x-1)$ $f(x+1) = 1 - f(x)$ $f(x+1) = 1 - (1 - f(x-1)) = f(x-1)$

16. B The new computer performs $\frac{x}{y}$ calculations per second, or $\frac{60tx}{y}$ calculations in t minutes.

The old computer performs $\left(\frac{r}{60s}\right)(60t)$ calculations in t minutes. Together they perform

$$\frac{60tx}{y} + \frac{60rt}{60s} = t\left(\frac{60x}{y} + \frac{r}{s}\right)$$

17. A. $A = |2-4i| = \sqrt{4+16} = \sqrt{20}$ $L = (1+i)^4 = -4$ $P = i^{2004} = 1$ $H = (2+i)(2-i) = 5$

$ALPHA = (\sqrt{20})(-4)(1)(5)(\sqrt{20}) = 20(-20) = -400$

18. A. $\cos(A+B) = \cos A \cos B - \sin A \sin B = \frac{-2\sqrt{2}}{3}\left(\frac{-3}{5}\right) - \frac{1}{3}\left(\frac{-4}{5}\right) = \frac{6\sqrt{2}}{15} + \frac{4}{15}$

19. D. Using the counting principle $\left(\underset{\text{5-9 are possible}}{5}\right)\left(\underset{\text{The 3}}{1}\right)\left(\underset{\text{Any digit except 3}}{9}\right)\left(\underset{\text{Any digit except 3}}{9}\right) = 405$

The 3 can be in the 2nd, 3rd and 4th positions so multiply by 3 and $3(405) = 1215$

20. B. M is on HA and U is on AT. Due to symmetry of the figure, triangle MAU is an isosceles right triangle. If we label the triangle sides as y and AM and AU as x, the following relationships are true. $2x^2 = y^2$ $(1-x)^2 + 1 + y^2$ $2x^2 = 1 - 2x + x^2 + 1$

$x^2 + 2x - 2 = 0$ and $x = -1 \pm \sqrt{3}$ only $x = -1 + \sqrt{3}$ works

$y = \sqrt{2}(-1+\sqrt{3}) = -\sqrt{2} + \sqrt{6}$ Area $= \frac{s^2\sqrt{3}}{4} = \frac{(-\sqrt{2}+\sqrt{6})^2\sqrt{3}}{4} = \frac{(8-4\sqrt{3})\sqrt{3}}{4} = 2\sqrt{3} - 3$

21. D. $3\log_x 5 \cdot \log_{25} x \cdot 4 = 12\log_{25} x \cdot \log_x 5 = 12\log_{25} 5 = 12\left(\frac{1}{2}\right) = 6$

22. A.

$x^2 - 4x + 1 = A(x^2+1) + (Bx+C)(x-3)$ if $x = 3 \Rightarrow$ 9-12+1=10A $\Rightarrow$ A=$\frac{-1}{5}$ if $x = 0 \Rightarrow$

$1 = \frac{-1}{5} + -3C \Rightarrow \frac{6}{5} = -3C \Rightarrow$ C=$\frac{-2}{5}$ if $x = 1 \Rightarrow -2 = \frac{-2}{5} + \left(B - \frac{2}{5}\right)(-2) \Rightarrow$

$-2 = \frac{-2}{5} - 2B + \frac{4}{5} \Rightarrow -2B = \frac{-12}{5} \Rightarrow B = \frac{6}{5}$ $\frac{-1}{5} - \frac{2}{5} + \frac{6}{5} = \frac{3}{5}$

23. D Since the coefficient of x^3 is 0, the sum of the roots is 0 and the 4th root is –4, the coefficient of x^2 is the sum of the combinations of the 4 roots taken 2 at a time.

A = $(-1)(2) + (-1)(3) + (-1)(-4) + (2)(3) + (2)(-4) + (3)(-4) = -15$

B = $-1((-1)(2)(3) + (-1)(2)(-4) + (-1)(3)(-4) + (2)(3)(-4)) = -1(-6+8+12-24) = (-1)(-10) = 10$

$C = (-1)(2)(3)(-4) = 24$ $2C - AB = 48 - (-15)(10) = 198$

24. A. $\lim_{n\to\infty}\left(\frac{5+n}{4+n}\right)^{100}\left(\frac{5^n}{5^{n+1}}\right) = (1)\left(\frac{1}{5}\right) = \frac{1}{5}$

25. B.

Substitution gives $r = \frac{3y}{r} \Rightarrow r^2 = 3y \Rightarrow x^2 + y^2 - 3y = 0 \Rightarrow x^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$.

The graph is a circle with radius of $\frac{3}{2}$ and therefore $C = 2\pi r = 2\pi\left(\frac{3}{2}\right) = 3\pi$

26. D. $\sin 2\theta = 2\sin\theta\cos\theta \quad \Rightarrow \quad \sin 2\theta = 2\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) = \frac{4\sqrt{5}}{9}$

27. D

Let r_1, r_2, r_3, and r_4 be the roots of the given polynomial. Then $g(x) = f(x-3)$ and $g(x+3) = f(x+3-3)$ and therefore $g(x) = (x-3)^4 + 3(x-3)^3 - 3(x-3)^2 + 4(x-3) - 6$

28. B

$x^2 + 2xy + y^2 = 16 \Rightarrow x^2 + 4 + y^2 = 16 \Rightarrow x^2 + y^2 = 12 \Rightarrow x^6 + 3x^4y^2 + 3x^2y^4 + y^6 = 1728$

$x^6 + y^6 = 1728 - 3x^2y^2(x^2 + y^2) = 1728 - (3)(4)(12) = 1728 - 144 = 1584$

29. B.

let r=the denominator. Then $r - 1 = 5\sqrt[3]{2} + 7\sqrt[3]{4}$ then $(r-1)^3 = r^3 - 3r^2 + 3r - 1 = \left(5\sqrt[3]{2} + 7\sqrt[3]{4}\right)^3 =$

$250 + 1050\sqrt[3]{2} + 1470\sqrt[3]{4} + 1372 = 1622 + 210\left(5\sqrt[3]{2} + 7\sqrt[3]{4}\right) \therefore r^3 - 3r^2 + 3r - 1 = 1622 + 210(r-1) =$

$1412 + 210r \Rightarrow r^3 - 3r^2 - 207r = 1413 \Rightarrow \frac{r^3 - 3r^2 - 207r}{1413} = 1 \quad \therefore \frac{1}{r} = \frac{r^2 - 3r - 207}{1413} = \frac{r(r-3) - 207}{1413} =$

$\frac{18\sqrt[3]{4} + 93\sqrt[3]{2} - 69}{1413} = \frac{6\sqrt[3]{4} + 31\sqrt[3]{2} - 23}{471}$ and $\frac{\sqrt[3]{2}}{r} = \frac{12 + 31\sqrt[3]{4} - 23\sqrt[3]{2}}{471}$

30. C.

$\log_2(x-16) = \frac{\log_2(x-4)}{\log_2 4} \Rightarrow \log_2(x-16) = \frac{1}{2}\log_2(x-4) \Rightarrow 2\log_2(x-16) = \log_2(x-4) \Rightarrow$

$\log_2(x-16)^2 = \log_2(x-4) \Rightarrow (x-16)^2 = (x-4) \Rightarrow x^2 - 32x + 256 = x - 4 \Rightarrow$

$x^2 - 33x + 260 = 0 \Rightarrow (x-13)(x-20) = 0$ only $x = 20$ works

TB 1. 4 $(x + 2)^x = x^x + x(x)^{x-1}(2) + \ldots$ $x^x + x(x)^{x-1}(2) = 768 \rightarrow x^x + 2x^x = 768 \rightarrow$
$3x^x = 768 \rightarrow x^x = 256 \rightarrow x = 4$

TB 2. $\frac{2}{n}$ Starting at n=3 $\frac{2}{3}\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)\ldots\frac{n-1}{n} = \frac{2}{n}$