Mu Alpha Theta National Convention: Denver, 2001
Individual Test – Alpha Division

1. Given that $f(x) = x^2 + 3x + 4$ and $g(x) = 2x + 3$, find $g(f(x))$.

   (A) $4x^3 + 6x^2 + 13x + 12$ (B) $2x^2 + 6x + 11$
   (C) $2x^3 + 9x^2 + 17x + 12$ (D) $4x^2 + 18x + 22$ (E) NOTA

2. Brian starts jogging at a constant rate of five miles per hour. Half an hour later, Keith starts running along the same route at seven miles per hour. For how many minutes must Keith run to catch Brian?

   (A) 75 minutes (B) 60 minutes (C) 150 minutes (D) 120 minutes (E) NOTA

3. If *a*, *b*, and *c* are all positive real numbers, simplify $\sqrt[5]{a^{12}b^{34}c^{56}}$.

   (A) $a^2b^6c^5\sqrt[5]{a^2b^4c^2}$ (B) $a^2b^6c^{11}\sqrt[5]{a^2b^4c}$
   (C) $a^2b^{17}c^{28}$ (D) $a^3b^7c^{12}\sqrt{a^3b^5c}$ (E) NOTA

4. Let $i = \sqrt{-1}$. Compute $(3 - 4i)(4 + i)$.

   (A) $16 - 13i$ (B) $8 + 19i$ (C) $5\sqrt{17}$ (D) $8 - 13i$ (E) NOTA

5. Given that *x* is the solution to the equation $\log(x - 1) = k$, where *k* is a whole number, let *y* be *x* when its digits are reversed. What is the ratio of *x* to *y*?

   (A) 1:2 (B) 2:1 (C) 1:1 (D) 1:3 (E) NOTA

6. One leg of a right triangle is two millimeters longer than twice the length of the other leg. The hypotenuse is one millimeter longer than the longer of two legs. What is the perimeter of the triangle, in millimeters?

   (A) 24 mm (B) 30 mm (C) 40 mm (D) 56 mm (E) NOTA

7. If *a* and *b* are the solutions to the equation $x^2 - 5x + 9 = 0$, what is the value of $(a - 2)(b - 2)$?

   (A) 5 (B) –9 (C) 3 (D) 23 (E) NOTA

8. Find the sum of all the *x*-coordinates of the points which trisect the line segment connecting $\left(\frac{1}{2},\frac{3}{4}\right)$ and $\left(\frac{5}{6},\frac{7}{8}\right)$.

(A) $\frac{13}{8}$ (B) $\frac{73}{24}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$ (E) NOTA

9. Evaluate: $\sum_{n=1}^{7}(5n+2)$

(A) 144 (B) 288 (C) 154 (D) 308 (E) NOTA

10. Which of the following angles is coterminal to $\frac{5063\pi}{6}$?

(A) $\frac{\pi}{6}$ (B) $\frac{11\pi}{6}$ (C) $\frac{7\pi}{6}$ (D) $\frac{5\pi}{6}$ (E) NOTA

11. For the final in Professor Richard's class, the average failing score was 66 and the average passing score was 81. The overall average for the forty students in the class was 75. How many students passed the final?

(A) 12 (B) 16 (C) 24 (D) 28 (E) NOTA

12. Determine all possible values for *x* on the interval $\frac{\pi}{4} \le x \le \frac{7\pi}{4}$ for which $\sin^2 x = 1$.

(A) $\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$ (B) $\frac{\pi}{2},\frac{3\pi}{2}$
(C) $\frac{\pi}{6},\frac{5\pi}{6}$ (D) $\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$ (E) NOTA

13. Out of Norris' fourteen Alphas, two must take the Gemini test and two others must take the Proofs test. How many ways can Norris assign his Alphas to these tasks?

(A) 5006 (B) 5206 (C) 6006 (D) 6206 (E) NOTA

14. How many values of *x* on the interval $0 < x < 2\pi$ satisfy $1 + \sec x = \tan x + \sin x$?

(A) 4 (B) 3 (C) 2 (D) 1 (E) NOTA

15. *ABCDEF* is a regular hexagon of side length 3. What is the ratio of the area common to ΔACE and ΔBDF to the area of *ABCDEF*?

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{2}{5}$ (D) $\frac{1}{3}$ (E) NOTA

16. Find the sine of the largest angle of a triangle having sides of 3, 7, and 8.

(A) $\frac{4\sqrt{3}}{7}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\frac{3}{21}$ (D) $\frac{13}{14}$ (E) NOTA

17. Given that $x^2 + y^2 = 6xy$ and $0 < y < x$, find $\frac{x+y}{x-y}$.

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) 1 (D) $\sqrt{2}$ (E) NOTA

18. How many real values of x on the interval (0, 1.57) are there for which $\left(\frac{\sin^2 x}{\tan^4 x}\right)^3 \left(\frac{\csc^3 x}{\cot^6 x}\right)^2 = 1$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

19. Which of the following is an equation of a degenerate conic?

I. $3x^2 - 6x + 2y^2 + 12y + 21 = 0$
II. $2x^2 - y^2 + 28x + 2y + 95 = 0$
III. $9x^2 - 90x - 4y^2 + 24y + 189 = 0$
IV. $2x^2 + 8x + 2y^2 - 24y + 80 = 0$

(A) I & II only (B) I & III only (C) I & IV only (D) I, III, & IV only (E) NOTA

20. Two vectors have magnitudes of 3 and 4 and the angle between them is $225°$. What is the dot product of the vectors?

(A) $6\sqrt{2}$ (B) $-12\sqrt{2}$ (C) $12\sqrt{2}$ (D) $-6\sqrt{2}$ (E) NOTA

21. Two hundred fifteen distinct numbers are picked from the set {1, 2, 3, … 320, 321}. Find the probability that at least three of those numbers are consecutive.

(A) $\frac{1}{4}$ (B) 1 (C) $\frac{2}{5}$ (D) $\frac{1}{3}$ (E) NOTA

22. Evaluate: $\cos^6 x + 3\cos^4 x \sin^2 x + 3\cos^2 x \sin^4 x + \sin^6 x$

(A) $\cos^2 x$ (B) $\cos^2 x - 1$ (C) $1 + \sin^2 x$ (D) 1 (E) NOTA

23. In ΔABC where $BC = 3$, $AC = 4$, and $AB = 5$, a circle is drawn so that it's tangent to AB at D and C lies on the circle. What is the area of the circle if CD is a diameter of the circle?

(A) $\frac{25\pi}{16}$ (B) 4π (C) $\frac{36\pi}{25}$ (D) $\frac{16\pi}{9}$ (E) NOTA

24. In how many distinct points do the polar graphs $r = 1$ and $r = 1 - 2\cos\theta$ intersect?

(A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

25. If $\alpha = \log_x 7$, $\beta = \log_x 3$, and $\gamma = \log_x 5$ for $x > 1$, find the maximum value of $f(a,b,c) = \log_x(ab^2c^3)$ where a, b, and c are distinct elements of the set {3, 5, 7}.

(A) 7α (B) $2\alpha + 3\gamma + \beta$ (C) $3\alpha + 2\gamma + \beta$ (D) $\alpha + 2\gamma + 3\beta$ (E) NOTA

26. Evaluate $\arctan\frac{8}{13} + \arctan\frac{1}{4} + \arctan\frac{5}{21} + \arctan\frac{3}{5}$, where $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$ for all real values of x.

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) 1 (D) $\frac{\pi}{8}$ (E) NOTA

27. What is the standard deviation of a data set consisting of thirty-six 4s and sixty-four 3s?

(A) $\frac{12}{25}$ (B) $\frac{9}{25}$ (C) $\frac{6}{25}$ (D) $\frac{3}{25}$ (E) NOTA

28. Evaluate: $\sin\left(\frac{\pi}{6}\right)+\cos\left(\frac{2\pi}{3}\right)-\tan\left(\frac{3\pi}{4}\right)$

(A) -1 (B) $-\frac{1}{2}$ (C) $\sqrt{3}$ (D) 1 (E) NOTA

29. Find the product of 865_9 and 71_9 in base 9.

(A) 68055_9 (B) 71375_9 (C) 61415_9 (D) 64735_9 (E) NOTA

30. Find the sum of all x on the interval $0 \le x \le 2\pi$ for which $\sin 3x = \cos 6x$.

(A) $\frac{5\pi}{2}$ (B) $\frac{9\pi}{2}$ (C) $\frac{13\pi}{2}$ (D) $\frac{17\pi}{2}$ (E) NOTA

31. Simplify $\sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}$.

(A) $\frac{e^x+e^{-x}}{2}$ (B) $\frac{1}{e^{2x}+e^{-2x}}$ (C) 2 (D) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$ (E) NOTA

32. If $\sin\theta=\frac{\sqrt{5}}{3}$, where $\frac{\pi}{2}<\theta<\pi$, what is $\cos 3\theta$?

(A) $\frac{9}{16}$ (B) $\frac{27}{86}$ (C) $\frac{22}{27}$ (D) $\frac{1}{9}$ (E) NOTA

33. Find the sum of the elements of the matrix product $\begin{bmatrix}7 & -1 & 3\\ 2 & 0 & 5\end{bmatrix}\begin{bmatrix}2\\4\\6\end{bmatrix}$.

(A) 17 (B) 62 (C) 0 (D) 192 (E) NOTA

34. Determine the sum of all the natural numbers less than 115 that are not divisible by 7.

(A) 5121 (B) 5603 (C) 6441 (D) 7393 (E) NOTA

35. For integer values of *n*, $(n^2+n)^3(2n+1)$ is always evenly divisible by which of the following?

(A) 42 (B) 36 (C) 24 (D) 21 (E) NOTA

36. Given that $\sin x + \cos x = \frac{6}{5}$ where $0 \le x \le \frac{\pi}{6}$, find the value of $\sin x$.

(A) $\frac{5-2\sqrt{3}}{10}$ (B) $\frac{6-\sqrt{13}}{20}$ (C) $\frac{6+\sqrt{13}}{20}$ (D) $\frac{6-\sqrt{14}}{10}$ (E) NOTA

37. Find the equation of the plane that contains the lines $\frac{1-x}{2} = y-4 = z$ and $\frac{2-x}{3} = \frac{y-1}{4} = 2-z$.

(A) $8x = 2y + 7z$ (B) $x + y + z = 5$
(C) $3x - y - 3z + 1 = 0$ (D) $2x + y + 6z = 6$ (E) NOTA

38. In a triangle with sides *a*, *b*, and *c*, $(a+b+c)(a+b-c) = 3ab$. Find $\sec C$, where *C* is the angle opposite side *c*.

(A) $\frac{3}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) 2 (D) $\frac{5}{4}$ (E) NOTA

39. Let *S* be the set of whole numbers less than 1136. How many members of *S*, when cubed, leave a remainder of 1 or 3 upon division by 4?

(A) 568 (B) 602 (C) 327 (D) 413 (E) NOTA

40. What is the shape of the conic section with equation $4x^2 - 6x + y^2 + 5y = 20$?

(A) circle (B) non-circular ellipse
(C) parabola (D) hyperbola (E) NOTA

41. Find the ordered pair (*x*, *y*) that satisfies the equations $\ln x - \ln y = \log_y x$ and $x^4 = y^9$.

(A) $\left(e^{1/2}, e^{1/3}\right)$ (B) $\left(e^{40/7}, e^{5/6}\right)$ (C) $\left(e^{81/20}, e^{9/5}\right)$ (D) $\left(e^{-4/3}, e^{18/5}\right)$ (E) NOTA

42. The sum of two numbers is 60, and their product is 62. What is the sum of the squares of the two numbers?

(A) -122 (B) $3662 + 62i$ (C) 122 (D) 3476 (E) NOTA

43. George has 162 coins in his collection of nickels, dimes, and quarters, which has a total value of $22.00. If George has twelve fewer nickels than quarters, how many dimes does he have? Assume George only began collecting recently, so none of his coins have appreciated in value.

(A) 70 (B) 72 (C) 73 (D) 74 (E) NOTA

44. If $u = 2\sin x + \cfrac{\cos^2 x}{2\sin x + \cfrac{\cos^2 x}{\ddots}}$, $0 < \cos x < \sin x$, and $\cos x = \dfrac{1}{3}$, what is the value of u?

(A) $\dfrac{2\sqrt{2}+3}{3}$ (B) $\dfrac{1+\sqrt{5}}{2}$ (C) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (D) $\sqrt{2}-1$ (E) NOTA

45. For what values of r is $\dfrac{2r}{r+3} < \dfrac{r}{r-3}$?

(A) $-3 < r < 9$ (B) $-3 < r < 0$ (C) $0 < r < 9$ (D) $0 < r < 3$ (E) NOTA

46. The sum of two numbers is 486, and the larger of the two numbers is 100 greater than the smaller of the two numbers. What is the smaller of the two numbers?

(A) 193 (B) 233 (C) 243 (D) 293 (E) NOTA

47. A right rectangular prism has faces with areas of 64, 80, and 20 square centimeters. What is the volume of this prism, in cubic centimeters?

(A) 720 cm^3 (B) 640 cm^3 (C) 360 cm^3 (D) 320 cm^3 (E) NOTA

48. N is an integer greater than 1 for which $\log(N) - [\log(N)] = 0$, where $[x]$ represents the greatest integer less than or equal to x. What percentage of natural numbers less than or equal to N are relatively prime to N?

(A) 32% (B) 60% (C) 40% (D) 55% (E) NOTA

49. The first term of an arithmetic sequence is 17, and the tenth term is 167. What is the common difference of the sequence?

(A) 15 (B) $\frac{29}{2}$ (C) 14 (D) $\frac{50}{3}$ (E) NOTA

50. Evaluate: $\sqrt{(\sqrt{2}+\sqrt{13}+\sqrt{17})(\sqrt{13}+\sqrt{17}-\sqrt{2})(\sqrt{2}+\sqrt{17}-\sqrt{13})(\sqrt{2}+\sqrt{13}-\sqrt{17})}$

(A) 16 (B) 14 (C) 12 (D) 10 (E) NOTA