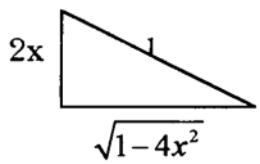
ALPHA INDIVIDUAL TEST--SOLUTIONS

1. C $x^9+9x^8y+36x^7y^2+\ldots$

2. D $\sum_{n=1}^{100}(2n+1)=3+5+7+\ldots+197+199+201=(204)(50)=10,200$

3. C Completing the square, we get: $4(x-4)^2+9(y+1)^2=36$ and the center is (4, -1).

4. C 

$\cos(\sin^{-1}2x)=\sqrt{1-4x^2}$

5. D $t_n=t_1+(n-1)d \Rightarrow t_{30}=-2+29(5)=143.$
$S_{30}=\frac{30}{2}(-2+143)=2115.$

6. A $70=-2+24d \Rightarrow d=3$. And, $t_{77}=-2+73d=217.$

7. B $36^2=16^2+26^2-2(16)(26)\cos\theta \Rightarrow \theta=116^\circ$

8. A $x^2=400^2+300^2-(400)(300)\cos 110^\circ \Rightarrow x=576.$

9. C $27^{2x-y}=\frac{1}{81} \Rightarrow 3^{6x-3y}=3^{-4} \Rightarrow 6x-3y=-4.$ Using this equation and using x= 3/2 from $\log_x\frac{729}{64}=6 \Rightarrow x^6=\frac{729}{64}$, yields y =13/3 and $x^2y=\frac{39}{4}$.

10. C Let the geometric sequence be a, ar, ar^2, ... and the arithmetic sequence be 0, d, 2d,... The sequence formed by summing corresponding terms is 1, 1, 2,...so a = 1. Using the 2nd and 3rd terms, we get r + d = 1 and r^2 + 2d = 2. Solving, we get r = 2 (since r $\neq 0$) and d = -1. The sum of the 1st 10 terms of the composite series is just equal to the sum of the sums of the 2 separate series, or $(0-1-2-\ldots-9)+(1+2+4+\ldots+2^9)$. The arithmetic part is the negative of the sum of the first 9 integers or –45. The geometric part is the sum of the first 10 powers of 2, which is 1023. The sum is 1023 – 45 = 978

Alpha Individual Test--Solutions

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11. B To find $y = f^{-1}(-2)$, we write $-2 = f(y) = \frac{1}{y+2} \Rightarrow y = \frac{-5}{2}$. This gives $g(f^{-1}(-2)) = g\left(\frac{-5}{2}\right) = 3$.

12. A From $f(x) = f(2a)^x$, we get $f(2a) = f(2a)^{2a}$.
Dividing by $f(2a)$ gives $1 = f(2a)^{2a-1}$ or $2a - 1 = 0$ and $a = \frac{1}{2}$.

13. D To multiply a 2 x 3 and a 4 x 2 matrix, we have to put the 4 x 2 matrix first. The dimension of the product is a 4 x 3 matrix with 12 elements.

C

14. Let $x = \sqrt[3]{5+2\sqrt{13}} + \sqrt[3]{5-2\sqrt{13}}$. Cubing both sides, we get:
$x^3 = 10 + 3\sqrt[3]{5+2\sqrt{13}}\sqrt[3]{5-2\sqrt{13}}\left(\sqrt[3]{5+2\sqrt{13}} + \sqrt[3]{5-2\sqrt{13}}\right)$ or $x^3 = 10 + 3\sqrt[3]{25-52}(x)$ or $x^3 + 9x - 10 = 0$. The only real root to this equation is $+1$.

15. B The coefficient of abc^2 is the same as the number of ways to arrange $abcc$, which is $\frac{4!}{1!1!2!} = 12$.

16. B This is the form of the identity: $\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$. Thus the sum is $100 \bullet 2^{99}$.

17. B The product is $\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{4}{5}\right)\cdots\left(\frac{n-1}{n}\right)$, which telescopes into $\frac{2}{n}$.

18. D The 4 countries can be ordered around the table in (4 – 1) ! = 6 ways. The Americans and Russians can be ordered (in their own group) in 3! = 6 ways. The Germans can be ordered in 4! = 24 ways and the French in 2. The number of possible seating is thus (6)(6)(6)(24)(2) = 10368.

ALPHA INDIVIDUAL TEST--SOLUTIONS

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19. E A line can intersect a hyperbola in 0, 1, or 2 points. Since we are given that the lines intersect the hyperbola, we can rule out 0. The 1 is not ruled out because you could have a line parallel to one of the asymptotes. It will meet the hyperbola in only one point. If it met the hyperbola in 2 points, it must then intersect the asymptote to which it is parallel, which is a contradiction. Since each line intersects the hyperbola in 1 or 2 points, the possible numbers of intersections are 2, 3 and 4.

20. C Using the form $y-k=\left(\frac{1}{4a}\right)(x-h)^2$ and substituting the vertex in for (h, k) and the point for (x, y), we get a = - ½ . So, $y=\frac{-x^2}{2}+4x-6$ and $abc=12$.

21. A Let the doubled roots be r and s. We then get 2r + 2s = 16 and $r^2+s^2+4rs=94$. Since $p=-(2r^2s+2rs^2)$ and $q=r^2s^2$, if we find rs and r + s, we find p and q. We already have r + s and we find rs by squaring r + s = 8. Subtracting this from the equation = to 94, we get rs = 15. Hence, p = -240 and q = 225, so p + q = -15.

22. C Let $a=2x$. Then, $f(a)=\frac{2}{2+a/2}$, so $2f(a)=\frac{4}{2+a/2}=\frac{8}{4+a}$. Thus, $2f(x)=\frac{8}{4+x}$.

23. B Let F = z = (r, θ). Hence, $\frac{1}{z}=\left(\frac{1}{r},-\theta\right)$. Since F is outside the unit circle, r > 1 and $\frac{1}{r}<1$. Thus, the reciprocal of z is inside the circle. Since the angle of $\frac{1}{z}$ is $-\theta$, it is on the opposite side of the x-axis from z. The only point that satisfies these conditions is C.

24. E $\cos\theta=\frac{\|(3\ \ 4\ \ 5)\|\|(-1\ \ 4\ \ 3)\|}{(3\ \ 4\ \ 5)\bullet(-1\ \ 4\ \ 3)}=\cancel{\frac{5\sqrt{13}}{14}}$. $\frac{14}{5\sqrt{13}}$

25. C This is most easily done by counting the wrong thing and subtracting. The number 2^{95} has 96 factors, all of which are powers of 2. The number of factors that are *smaller* than 1,000,000 may be found by finding the largest one that is smaller. We know $2^{10}=1024>10^3$, so $2^{20}>10^6=1,000,000$. But, $2^{19}=1024\bullet512$ is clearly smaller than 1,000,000, so this is the largest. The number smaller than 1 million is thus 19 + 1 = 20, and there are 96 – 20 = 76 greater than a million.

26. B In the given plane, each pair of lines may intersect in a point, for $\binom{5}{2}$ = 10 intersections.

Each line can intersect both circles in 2 points each, for $5 \cdot 2 \cdot 2 = 20$ intersections. Last, the 2 circles can intersect each other in 2 points = 2 intersections. The total is 32.

27. B The 9 one-digit numbers each contribute their 1, for a total of 9 digits. This leaves 852 – 9 = 843 to go. The 90 two-digit numbers each contribute 2 digits, for a total of 180 digits. This leaves 843 – 180 = 663 to go. The 900 three-digit numbers contribute 3 digits each, for a total of 2700 digits (this is over the limit of 852). Thus, all the remaining 663 are three-digit numbers, so 221 (663/3) are used. Adding these 221to the last two-digit, 99, so that the last number used is 99 + 221 = 320.

28. A Such a function changes sign when we reflect across the y-axis. The function thus flips over on reflection. Such a function has point symmetry with respect to the origin.

29. B If my dog walks until the rope pulls taut, she will get to the boundary of her roaming area. This boundary is an ellipse, since the sum of the distances from any point on the boundary to the stakes is the length of the rope, 30 feet. The stakes correspond to the foci and the rope to the constant sum of the distances. Hence, the major axis = 30. If we let the stakes lie on the x-axis and the midpoint of the line connecting the stakes be the origin, we get the equation to be $\frac{x^2}{225}+\frac{y^2}{b^2}=1$. We find b by noting that the distance from a focus to the center is c = 5 since the stakes are 10 feet apart, and b = $10\sqrt{2}$. The area of the roaming region is the area of the ellipse (abπ), which is $150\pi\sqrt{2}$.

30. B Multiplying both sides by r to create r^2, $r\cos\theta$, and $r\sin\theta$ terms, we get $r^2 = ar\sin\theta + br\cos\theta$. Changing to rectangular for, we then get $x^2 + y^2 = ay + bx$. This form describes a circle.