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1999 Mu Alpha Theta National Convention
Individual
Alpha

1. Compute $\log 2^{4^2} + \log 5^{2^4}$

a. 4 b. 8 c. 16 d. 32 e. not given

2. Until recently telephone numbers began with three-digit area codes governed by the following rules:
   (1) The first digit cannot be 0 or 1.
   (2) The second digit must be 0 or 1.
   (3) The third digit can be any of the ten digits.

   Rule 2 has now been changed so that the second digit can be any of the ten digits. How many additional area codes have been created by this change in the rules?

a. 160 b. 800 c. 22,560 d. 40,320 e. not given

3. Given convex Pentagon ABCDE whose coordinates are A(-2, 0), C(0, 10), E(3, 0), D(5, 6) and B(-3, 4). How many diagonals with positive slope can be drawn in Pentagon ABCDE?

a. 5 b. 4 c. 3 d. 2 e. not given

4. The solution set of $2x^2 - 3 < x$ is the set of all real numbers x such that

a. $0 < x < 1.5$ b. $x < 1.5$ or $x < -1$ c. $-1.5 < x < 1$
d. $-1 < x < 1.5$ e. not given

5. If $x - 1$, x, $x - 1$ is a geometric sequence, then $x =$

a. $\frac{-1}{2}$ b. -1 c. $\frac{1}{2}$ d. 1 e. not given

6. If $15^4 \bullet 6^5 \bullet 4^6 = 2^A \bullet 3^B \bullet 5^C$, then compute the value of A + B - C

a. 19 b. 22 c. 26 d. 30 e. not given

7 Compute $1^2 - 2^2 + 3^2 - 4^2 + \ldots - 1998^2 + 1999^2$

a. 1,999,000 b. 3,996,001 c. 3,994,002 d. 1999 e. not given

8. If $0^\circ < x < 90^\circ$, and $\cos x = \frac{3}{\sqrt{10}}$, compute $\log \sin x + \log \cos x + \log \tan x$.

a. 10 b. 1 c. -1 d. 10 e. not given

9. Find the sum of all positive values of t less than 180° that satisfy $2\sin^2 t - \cos t - 2 = 0$

a. 120° b. 150° c. 180° d. 210° e. not given

10. Compute the number of squares between 7^4 and 4^7.

a. 78 b. 79 c. 80 d. 81 e. not given

11. Compute the numerical value of $\log\left(\frac{2a}{b}\right) + \log\left(\frac{2b}{c}\right) + \log\left(\frac{5c}{d}\right) + \log\left(\frac{5d}{a}\right) + \log 1$

a. -1 b. 1 c. 2 d. 10 e. not given

12. The lengths of the three altitudes of a right triangle are 12, 15 and 20. What is the length of the hypotenuse?

a. 39 b. 34 c. 26 d. 25 e. not given

13. Given an ordinary six-sided die with the digits 1 - 6 on the faces, one digit per face. What is the probability that Kurt can toss this six-sided die, three times and get exactly one five?

a. $\frac{25}{216}$ b. $\frac{25}{144}$ c. $\frac{25}{72}$ d. $\frac{25}{36}$ e. not given

14. Simplify $\frac{\sin 2x}{1+\cos 2x}$ to a single trigonometric function of x.

a. cot x b. csc x c. tan x d. cot x e. not given

15. If $\log x \geq \log 2 + \frac{1}{2}\log x$,

a. $x \geq 2$ b. $x \leq 2$ c. $x \leq 4$ d. $x \geq 4$ e. not given

16. If $\cos(5n - 30)^\circ = \sin 50^\circ$ and $0^\circ < n < 90^\circ$ then n, in degrees, is what?

a. 2 b. 14 c. 16 d. 76 e. not given

17. A car travels at 48 miles per hour and gets 30 miles per gallon of gas. How long will it take, in minutes, to use two-thirds of a gallon of gas?

a. 37.5 b. 25 c. 22.5 d. 18 e. not given

18. When multiplied out, 17! is equal to 35568B428096000. Compute the missing digit B.

a. 1 b. 3 c. 5 d. 7 e. not given

19. In right triangle, PRQ with Angle R = 90°. RH and RM are the altitude and median to the hypotenuse respectively. If angle Q = 32°, find the measure of Angle HRM.

a. 24° b. 26° c. 42° d. 58° e. not given

20. Given that f is a linear function where $f(0) = 20$ and $f(4) = 0$, compute the numerical value for $f(1999)$.

a. -3998 b. -7976 c. -9975 d. -10,015 e. not given

21. Let x = .12345678910111213...998999, where the digits are obtained by writing the integers 1 through 999 in order. What is the 2001th digit to the right of the decimal point.

a. 0 b. 1 c. 2 d. 3 e. not given

22. How many consecutive zeros are at the end of the product of

$$15 \bullet 16 \bullet 17 \bullet 18 \bullet \ldots \bullet 98 \bullet 99 \bullet 100$$

a. 22 b. 23 c. 24 d. 25 e. not given

23. How many distinct triangles can be formed by connecting three different vertices of a cube?

a. 24 b. 56 c. 72 d. 336 e. not given

24. Given the following arithmetic sequence:

$$3y - 1,\ 4x - 1,\ z,\ y + 4,\ x + 2,\ t,\ \frac{-x}{2}$$

Find the sum of t, x, y and z.

a. 59 b. 48 c. 37 d. 26 e. not given

25. The sequence $\frac{1}{2}, \frac{5}{3}, \frac{11}{8}, \frac{27}{19}, \ldots$ is formed as follows: each denominator is the sum of the numerator and denominator of the previous term; each numerator is the sum of its own denominator and the previous denominator. The successive terms are approaching the real number n as a limit. Compute n.

a. $\sqrt{2}$ b. e c. π d. $+\infty$ e. not given

26. The successive sides of a quadrilateral are 2, 6, 9 and x. If the diagonals of the quadrilateral are perpendicular, compute x.

a. 5 b. 7 c. 11 d. 13 e. not given

27. The length of a common internal tangent to two circles is 7, and a common external tangent is 11. Compute the product of the radii of the two circles.

a. 24 b. 22 c. 21 d. 18 e. not given

28. In square ABCD let M be the midpoint of side AB. Determine the sine of Angle MDB if MD = $\frac{\sqrt{5}}{2}$ and BD = $\sqrt{2}$. (Note: When labeling the vertices of this square, start in the upper left hand corner, go in a clockwise fashion, using successive letters. Thus, D will be located in the lower, left-hand corner.)

a. $\frac{\sqrt{10}}{10}$ b. $\frac{\sqrt{10}}{4}$ c. $\frac{\sqrt{5}}{4}$ d. $\frac{\sqrt{5}}{10}$ e. not given

29. For how many values of x will $x^{(x+1)^2} = x^{16}$?

a. 3 b. 4 c. 5 d. 6 e. not given

30. How many base 4, four-digit numbers when written in base 9 will be three digit numbers?

a. 333 b. 316 c. 255 d. 175 e. not given

Tie Breakers:

T1. If g (x - 1) = $x^2 + 2$, then compute g (19)

T2. Find the smallest positive value of x which satisfies $\sin x \bullet \sin 40° - \cos x \bullet \cos 40° = \frac{1}{2}$ for all for values of x such that $180° \le x \le 360°$

T3. If three non-collinear points determine a plane, how many planes are determined by ten points, no three of which are collinear?