ALPHA INDIVIDUAL 1994

1.) C $\log 2^{16} + \log 5^{16}$

$16 \log 2 + 16 \log 5$

$16(\log 2 + \log 5)$

$16(\log 10)$

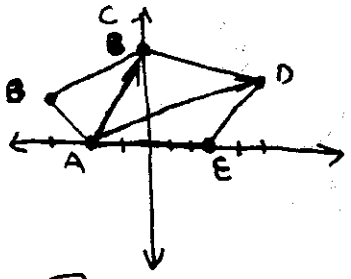
(16)

2. E $8 \cdot 2 \cdot 10 = 160$ 800

$8 \cdot 10 \cdot 10 = 800$ −160

640

3. D



2 diagonals HAVE A POSITIVE m.

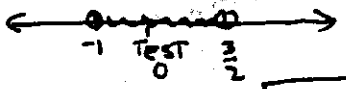
4. D $2x^2 - x - 3 < 0$

$(2x-3)(x+1) < 0$

$2x-3=0$ $x+1=0$

$2x=3$

$x=\frac{3}{2}$ or $x=-1$



$-1 < x < 1.5$

5. C $\frac{x}{x-1} = \frac{x-1}{x}$

$x^2 = x^2 - 2x + 1$

$2x = 1$

$x = \frac{1}{2}$

6. B $15^4 \cdot 6^5 \cdot 4^6 = 3^4 \cdot 5^4 \cdot 2^5 \cdot 3^5 \cdot 2^{12}$

$= 2^{17} \cdot 3^9 \cdot 5^4$

$A=17,\ B=9,\ C=4$

$17+9-4 = 22$

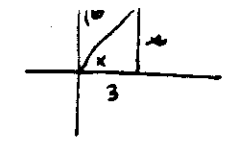
7. A $S = 1999^2 - 1998^2 + 1997^2 - 1996^2 + \ldots + 3^2 - 2^2 + 1^2$

$\Delta = (1999-1998)(1999+1998) + (1997+1996)(1997-1996) \ldots$

$\Delta = 1999 + 1998 + 1997 + 1996 + \ldots + 3 + 2 + 1$

$\Delta = \frac{1999 \cdot 2000}{2} = 1{,}999{,}000$

8. C



$10 = 9 + \Delta^2$

$1 = \Delta^2$

$1 = \Delta$

$\log \sin x + \log \cos x + \log \tan x$

$\log (\sin x \cdot \cos x \cdot \tan x)$

$\log (\sin x \cdot \cos x \cdot \frac{\sin x}{\cos x})$

$\log \sin^2 x$

$\log (\frac{1}{10})^2$

$\log \frac{1}{10}$ (−1)

$\log 10^{-1}$

9. D $2\sin^2 t - \cos t - 2 = 0$

$2(1-\cos^2 t) - \cos t - 2 = 0$

$2 - 2\cos^2 t - \cos t - 2 = 0$

$2\cos^2 t + \cos t = 0$

$\cos t (2\cos t + 1) = 0$

$\cos t = 0$ $t = 90°$

$\cos t = -\frac{1}{2}$ $t = 120°$

$90 + 120$

210

10. A 7^4 4^7

$(7^2)^2$ $(2^2)^7$

49^2 $(2^7)^2$

128^2

$128 - 49 - 1$

(78)

11. C $\log \left(\frac{2a}{b} \cdot \frac{2b}{c} \cdot \frac{5c}{d} \cdot \frac{5d}{a} \cdot 1\right)$

$\log (2 \cdot 2 \cdot 5 \cdot 5)$

$\log 100$

(2)

12. D 12 is the length of the altitude to the hypotenuse.

Thus, we have a (15, 20, 25) right Δ.

13. C $3 \cdot \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = 3\left(\frac{25}{216}\right) = \frac{25}{72}$

14. C $\frac{2\sin x \cos x}{1 + 2\cos^2 x - 1} = \frac{2\sin x \cos x}{2\cos^2 x} = \frac{\sin x}{\cos x} = \tan x$

15. D $\log x \geq \log z + \log x^{\frac{1}{2}}$

$\log x - \log x^{\frac{1}{2}} \geq \log z$

$\log \left(\frac{x}{x^{\frac{1}{2}}}\right) \geq \log z$

$x^{\frac{1}{2}} \geq z$

$x \geq 4$

16.) $\cos(5n-30)^\circ = \sin 50^\circ$

B $5n-30+50=90$

$5n+20=90$

$5n=70$

$\boxed{n=14}$

17. B $\frac{48}{60} = \frac{30}{x}$

$48x = 1800$

$x = 37.5$

$\frac{2}{3}(37.5)$

$2(12.5)$

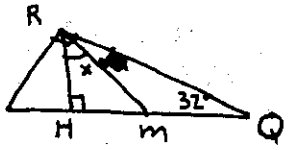
$\boxed{25}$

18. 355688428096000 $\Rightarrow$ 56+B is a multiple of 9.

D $56+B=63$

$B=7$

19. B



$\angle PQR = 90^\circ$

$\angle Q = 32^\circ$

$\angle P = 180^\circ - 90^\circ - 32^\circ = 58^\circ$

$\angle PRH = 90^\circ - 58^\circ = 32^\circ$

In $\triangle RMQ$, $RM = MQ$ thus, $\angle MRQ = 32^\circ$

$\angle HRM = 90^\circ - 32^\circ - 32^\circ = \boxed{26^\circ}$

20. C $y = mx+b$

$20 = m(0)+b$

$20 = b$

$y = mx+20$

$0 = 4m+20$

$-20 = 4m$

$-5 = m$

$f(x) = -5x+20$

$f(1999) = -5(1999)+20$

$= -9995+20$

$= -9975$

$f(1999) = \boxed{-9975}$

21. D $2001-9 = 1992$ for one digit #s

$1992-180 = 1812$ for two digit #s

$1812/3 = 604$ for 3 digit #s

$604-1 = 603$

2001st digit is $\boxed{3}$.

22. A The product of all integers between 1 and 100 has 24 factors of 5, thus there are 24 zeros. Subtract 2 from 24 for the 2-5's

(22)

23. B $_8C_3 = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \boxed{56}$

24. B

2nd term	5th term	7th term
$4x-1$	$x+2$	$-\frac{x}{2}$

$4x-1+3d = x+2$

$3d = -3x+3$

$d = -x+1$

$d = -8+1$

$d = -7$

$x+2+2d = -\frac{x}{2}$

$2x+4+4d = -x$

$4d = -3x-4$

$4(-x+1) = -3x-4$

$-4x+4 = -3x-4$

$-x = -8$

$x = 8$

1st + 24 cont.	2nd	3rd	4th
$3y-1 = 38-(-1)$	$4x-1$	$z = 31-7$	$y+4$
$3y-1 = 38$	$32-1$	$\boxed{z=24}$	$13+4$
$3y = 39$	31		(17)
$\boxed{y=13}$			

5th	6th	7th
$x+z = 17-7$	$t = 10-7$	$-\frac{x}{2} = 3-7$
$x+z = 10$	$\boxed{t=3}$	$-\frac{x}{2} = -4$
$x+8$		$-x = -8$
		$x = 8$

$x+t+z+y$

$8+3+24+13$

148 (48)

25. A $\frac{a}{b} = \frac{a+2b}{a+b}$

$a^2+ab = ab+2b^2$

$a^2 = 2b^2$

$\frac{a^2}{b^2} = 2 \Rightarrow \frac{a}{b} = \boxed{\sqrt{2}}$

26. B $2^2+9^2 = 6^2+x^2$

$4+81 = 36+x^2$

$85 = 36+x^2$

$49 = x^2$

$7 = x$

27. D $(R+r)^2+49 = d^2$ $(R-r)^2+121 = d^2$

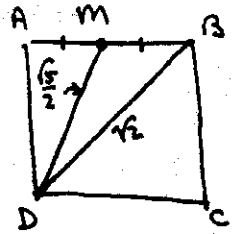
$(R+r)^2+49 = (R-r)^2+121$

$R^2+2Rr+r^2+49 = R^2-2Rr+r^2+121$

$4Rr = 72$

$\boxed{Rr = 18}$

28. A



Area of square = 1·1

Area $\triangle MDB = \frac{1}{4}(1) = \frac{1}{4}$

$\frac{1}{2}ab\sin C = \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\sqrt{2}$

$\sin\angle MDB = \frac{1}{4}$

$\frac{\sqrt{10}}{4}\sin\angle MDB = \frac{1}{4}$

$\sin\angle MDB = \frac{1}{4}\cdot\frac{4}{\sqrt{10}}$

$\sin\angle MDB = \frac{1}{\sqrt{10}}$

$\sin\angle MDB = \boxed{\frac{\sqrt{10}}{10}}$

29. C $x^{(x+1)^2} = x^{16}$

$x^2+2x+1 = 16$

$x^2+2x-15 = 0$

$(x+5)(x-3) = 0$

$x = -5$ or $x = 3$

YET. TRY THE TRADITIONAL 1, 0, and -1 They, too, work.

THERE ARE 5 VALUES.

30. D

Smallest 4 digit base 4 # 1000_4
largest " " " " 3333_4
Smallest 3 digit base 9 # 100_9
largest 3 digit base 9 # 888_9

$100_9 = 1101_4 = 81_{10}$

$3333_4 = 313_9 = 255_{10}$

$255 - 81 + 1 = 174 + 1 = \boxed{175}$

T1. $g(x-1) = x^2 + 2$ LET $R = x-1$

$x = R+1$

$g(r) = r^2 + 2$

$g(r+1) = (1+r)^2 + 2$

$= 1 + 2r + r^2 + 2$

$= r^2 + 2r + 3$

$g(19) = \boxed{402}$

T2 $\sin x \sin 40^\circ - \cos x \cos 40^\circ = \frac{1}{2}$

$\cos(x + 40^\circ) = \cos 300^\circ$

$x + 40 = 300$

$x = 260$

T3 $_{10}C_3 = \frac{10!}{(10-3)!\,3!} = \frac{10!}{7!\,3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$

$\boxed{120}$