ALPHA INDIVIDUAL TEST
1992 MU ALPHA THETA NATIONAL CONVENTION

1. An ellipse with center at (0,0) is inscribed in a rectangle. If the area of the ellipse is 15π and the semimajor axis is 5, what is the perimeter of the rectangle?

a) 15 b) 16 c) 30 d) 32 e) nota

2. If $\sin x = \cos 2x$ and $0 \le x \le \pi/2$, then $x = ?$

a) 0 b) $\pi/6$ c) $\pi/4$ d) $\pi/3$ e) nota

3. If, for all x, $f(x) = f(2a)^x$ and $f(x+3) = 27\, f(x)$, then a=?

a) 3 b) -3 c) 1 d) 6 e) nota

4. If P(A) = .2, (the probability of event A is 1/5), P(B) = .6, and $P(A \cap B) = .1$, find $P(A \cup B)$.

a) .08 b) .7 c) .8 d) .9 e) nota

5. Arccos(sin(Arccot(sec 0))) = ?

a) 0 b) $\pi/6$ c) $\pi/4$ d) $\pi/3$ e) nota

6. What type of figure does the graph of the set of pairs (x,y), where $x = \cos\phi$, $y = \sin\phi$, and $0 \le \phi < 2\pi$ turn out to be?

a) cardioid b) circle c) ellipse d) hyperbola e) nota

7. $$\frac{(n-2)!}{n!} - \frac{(n-1)(n-2)!}{n(n-1)!(n-1)} = \ ?$$

a) $\frac{1-n}{n(n-1)}$ b) 0 c) $\frac{n-2}{n}$ d) $\frac{n^2-4n+4}{n(n-1)}$ e) nota

8. Determine the coordinates of the foci of the conic with the equation:
$(x-2)^2 + (y-2)^2 = 1$.

a) $(\pm\sqrt{2}, 0)$ b) (0,1) & (1,0) c) $(\pm 2,0)$ d) $(2\pm\sqrt{2}, 2)$ e) nota

9. Find x if the determinant of the matrix A is 2x - 4, where

$$A = \begin{vmatrix} x-2 & 0 \\ 0 & x-3 \end{vmatrix}$$

a) x=2, x=5 b) x=1, x= -1 c) x=0, x=2 d) x=2, x=3 e) nota

10. Determine the value of x in the expression below:

$$\log_x x^{x^2} + \log_x x^{-5x} = \log_x\left(1/x^6\right)$$

a) x=2, x=1 b) x=2, x=3 c) x=4 d) x=3, x=1 e) nota

11. How many four-digit numbers are there such that the first digit is odd, the second is even, and there are no repetition of digits?

a) 1200 b) 1625 c) 200 d) 1400 e) nota

12. A standard deck of cards is cut into 2 piles. The first pile contains 7 times as many black cards as red cards. The second pile contains the number of red cards that is an exact multiple of the number of black cards. How many cards are in the first pile?

a) 36 b) 8 c) 24 d) 16 e) nota

13. To what real number does the sequence $\sqrt{6}$, $\sqrt{6-\sqrt{6}}$, $\sqrt{6-\sqrt{6-\sqrt{6}}}$ ... converge?

a) 3 b) 2 c) 5 d) 6 e) nota

14. If $4\sin^2 x + 11\sin x - 3 = 0$, then what is (are) the value(s) of $\log_2(\sin x)$?

a) 1/4 b) 2 c) -3 d) -2 e) nota

15. Let f be the function from real numbers to real numbers defined by:

$$f(x) = \begin{cases} x+2 \text{ , if 3 is a divisor of} [x] \\ x-1 \text{ , otherwise} \end{cases}$$

As usual, $[x]$ stands for the greatest integer function. Compute $f(f(f(f(f(\pi)))))$.

a) π b) $\pi + 1$ c) $\pi + 2$ d) $\pi + 3$ e) nota

16. What is the sum of the first 1992 terms of the series 1+2+3+4-4-3-2-1+..., if this first group of eight terms repeats indefinitely?

a) 0 b) 1 c) 4 d) 8 e) nota

17. Two parallel planes that are 3 units apart intersect a right circular cone and are perpendicular to its axis of symmetry. The intersection of each plane and the cone is a circle, one with radius 6 units and one with radius 4 units. Find the volume of the solid bounded by the 2 planes and the cone.

a) 228π b) 96π c) 324π d) 76π e) nota

18. If x and y are acute angles, such that $\tan x + \tan y = \sqrt{3}\,(1 - \tan x \tan y)$, and $\sqrt{2} \sin y = 1$, find x in degrees.

a) 60 b) 45 c) 30 d) 15 e) nota

19. Solve the following system for (x, y): $\cos^{-1} 1 = x - 2y$
$\tan^{-1} 1 = 2x - y$

a) $(0, \pi/4)$ b) $(\pi/6, 0)$ c) $(0, \pi/12)$ d) $(\pi/6, \pi/12)$ e) nota

20. A ball of radius R is tangent to the floor and one wall of a room. Find in terms of R, the radius of the largest sphere that can be inserted in the space between the ball, the wall and the floor.

a) $R\sqrt{2}$ b) $R(\sqrt{2} - 1)$ c) $R(3-2\sqrt{2})$ d) $R(\sqrt{2} + 1)$ e) nota

21. Each edge of a regular tetrahedron ABCD has length 12 inches. An ant crawls on the faces of the tetrahedron from the midpoint M, of edge AB to the centroid F of face BDC. What is the length of the shortest path?

a) $2\sqrt{21}$ b) $4\sqrt{3}$ c) 6 d) 12 e) nota

22. Find the sum of the infinite series: $\frac{1}{2} + \frac{2}{6} + \frac{3}{24} + \frac{4}{120} + \ldots$

a) 1 b) e c) π d) 4 e) nota

23. Find the horizontal asymptote (if it exists) for $y = x^{1/x}$, $x>0$.

a) There is none. b) $y = 1$ c) $y = 0$ d) $y = -1$ e) nota

24. If sin x + cos x = $\frac{-1}{5}$ and if $\frac{3\pi}{4} \le x \le \pi$, find the value of cos 2x.

a) 1/25 b) -24/25 c) 7/25 d) 24/25 e) nota

25. If $(1 + \cos 60° + i \sin 60°)^{12} = a + bi$, find the value of b.

a) 1 b) 0 c) -1 d) 2 e) nota

26. As a circle increases in size, the instantaneous rate of change of the area and radius are different except for one value of the radius. Which one of the following intervals contains the value of the radius for which the instantaneous rates of change of the area and radius are equal?

a) $0 \le r < .1$ b) $.1 \le r < .2$ c) $.2 \le r < .3$ d) $.3 \le r < .4$ e) nota

27. Let $f(x) = -x + x\ln x$, find $D_x[f^{-1}(0)]$.

a) 0 b) 1 c) e d) 1/e e) nota

28. If f is continuous for all reals, g(x) is a differentiable function and F(x) = $\int_0^{g(x)} f(t)\,dt$,

then F '(1) is:

a) f(g(1)) b) f(1) g' (1) c) f ' (g(1)) g' (1) d) f(g(1)) g'(1) e) nota

29. A water tank is to be drained for cleaning. If G(t) represents the number of gallons of water in the tank after the tank has been draining for t minutes, and $G(t) = 20(30 - t)^2$, then find t_0, if the average rate at which the water flows out between t_0 and $2t_0$ is 60 gallons/minute.

a) 10 min. b) 20 min. c) 30 min. d) 40 min. e) nota

30. Let $g(x) = e^{-x^2}$ and determine which one of the following statements is true:

a) g is a decreasing function b) g is an odd function

c) g is symmetric with respect to the x-axis d) (.5, $e^{-.25}$) is a point of inflection

e) nota