# 1992 National Mu Alpha Theta Convention

Alpha Individual Test Answers:

| | |
|---|---|
| 1. D | 16. A |
| 2. B | 17. D |
| 3. E | 18. D |
| 4. B | 19. D |
| 5. C | 20. C |
| 6. B | 21. A |
| 7. B | 22. A |
| 8. D | 23. B |
| 9. A | 24. C |
| 10. B | 25. B |
| 11. D | 26. B |
| 12. D | 27. B |
| 13. B | 28. D |
| 14. D | 29. E |
| 15. B | 30. E |

10

# 1992 National Mu Alpha Theta Convention

Alpha Individual Test Solutions:

1. D  $A = 15\pi = \pi ab$
$ab = 15$; given: $a = 5$
so: $b = 3$
and perimeter = 32.

2. B  In first quadrant, $\sin x = \cos(90 - x)$
or, $\cos 2x = \cos(90 - x)$
$2x = 90 - x \Rightarrow x = 30°$

3. E  $f(x) = (2a)^x$; $f(x+3) = (2a)^{x+3} = 27f(x)$
$(2a)^{x+3} = 27(2a)^x$
$(2a)^3 = 27$
$2a = 3 \Rightarrow a = 3/2$

4. B  $P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$
$= .2 + .6 - .1 = .7$

5. C  $\text{Arccos}(\sin(\text{Arccot}(\sec 0)))$
$= \text{Arccos}(\sin(\text{Arccot } 1))$
$= \text{Arccos}(\sin \pi/4)$
$= \text{Arccos}(1/\sqrt{2}) = \pi/4$

6. B  $x = \cos\phi$, $y = \sin\phi$
$x^2 + y^2 = \cos^2\phi + \sin^2\phi = 1$
circle with center (0,0) and r = 1.

7. B
$$\frac{(n-2)!}{n!} - \frac{(n-2)(n-2)!}{n(n-1)!(n-1)}$$
$$= \frac{(n-2)!}{n(n-1)(n-2)!} - \frac{(n-1)!}{n(n-1)(n-1)!}$$
$$= \frac{1}{n(n-1)} - \frac{1}{n(n-1)} = 0$$

8. D  center @ (2,2); a = 1, b = 1
$c = \sqrt{a^2 + b^2} = \sqrt{2}$
$F = (h \pm c, k) = (2 \pm \sqrt{2}, 2)$

9. A  $\det A = (x-2)(x-3) - 0$
$= x^2 - 5x + 6$
$x^2 - 5x + 6 = 2x - 4$
$x = 2, 5$

10. On next page

11. D  5 poss. for 1st digit(1,3,5,7,9)
5 poss. for 2nd digit(2,4,6,8,0)
8 poss. for 3rd digit(10-2)
7 poss. for 4th digit(10-3)
So, 5x5x8x7 = 1400

12. D  1st pick has b black & r red cards. b=7r, so r = 1,2, or 3. If r = 1, b = 7 and 2nd pile has 19 black & 25 red cards. If r = 2, b = 14 & 2nd pile has 12 black & 24 red cards, as needed. So, r = 1 & r = 3 are eliminated, and 1st pile has 16 cards.

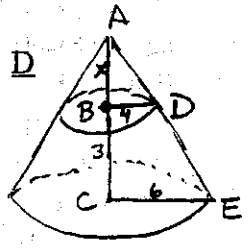
10. B $\log_x x^{x^2} = x^2$ and $\log_x x^{-5x} = -5x$
and $\log_x\left(1/x^6\right) = -6$
*So*, $x^2 - 5x = -6$ and $x = 2,3$

13. B Let S = Limit of this sequence.
$S = \sqrt{6-S} \Rightarrow S^2 = 6 - S$ and S = 2, -3
But, -3 is eliminated & 2 is the only soln.

14. D Let $y = \sin x \Rightarrow 4y^2 + 11y - 3 = 0$
and y = -3 or 1/4; -3 does not work, so:
$\log_2 y = \log_2\left(1/4\right) = -2$.

15. B Note that f(f(f(x))) = x, so f(f(f(f(f(π))))) = f(f(π))
= f(π + 2) = π + 1

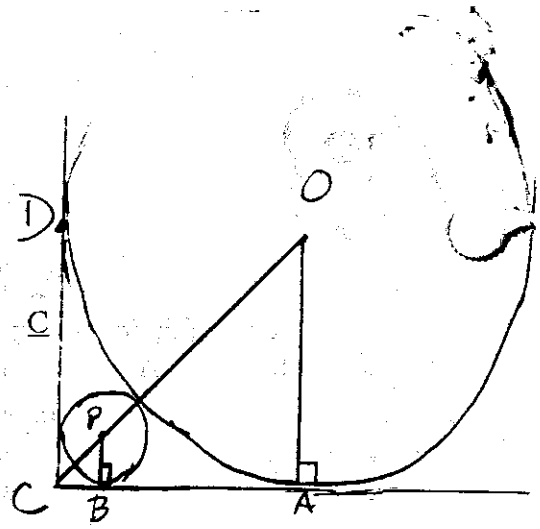
16. A Each group of 8 terms has a sum of 0.
Thus, 1992/8 has remainder of 0 and sum is 0.

17. D


$\frac{x}{4} = \frac{x+3}{6}$ $(\Delta ABD \approx \Delta ACE) \Rightarrow x = 6$
$V = \frac{1}{3}\pi \bullet 6^2 \bullet 9 - \frac{1}{3}\pi \bullet 4^2 \bullet 6 = 76\pi$

18. D Notice that (tan x + tan y)/(1- tan x tan y) = $\sqrt{3}$
= tan (x+y) ⇒x+y = 60°. Also, $\sqrt{2}$ sin y = 1,
so y = 45° and x = 15°.

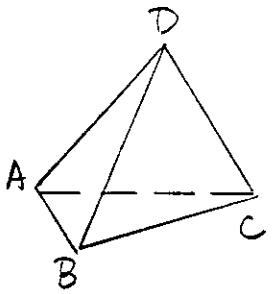
19. D Since $\text{Cos}^{-1} = 0$ and $\text{Tan}^{-1} = \frac{\pi}{4}$, the 2 equations
become x - 2y = 0 and $2x - y = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{6}, y = \frac{\pi}{12}$

20. C


OA = R and OC = $R\sqrt{2}$
r = PB, so PC = $r\sqrt{2}$
Thus, $R\sqrt{2}$ = OT+TP+PC
= R + r + $r\sqrt{2}$

$R\sqrt{2} - R = r + r\sqrt{2}$ and
$r = \frac{R(\sqrt{2}-1)}{\sqrt{2}+1} = R\left(3 - 2\sqrt{2}\right)$

21. A

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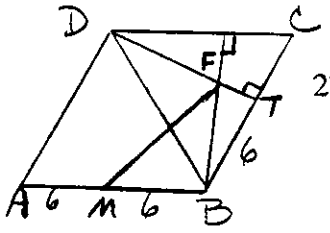


Shortest path is MF. $m\angle DBF = 30°$, $m\angle MBF = 90°$
If BT = 6, then BF = $2\bullet\frac{6}{\sqrt{3}} = 4\sqrt{3}$.

So, MF = $\sqrt{6^2 + \left(4\sqrt{3}\right)^2} = 2\sqrt{21}$.

27. B $f' = \ln x$;
$D_x(f^{-1}(x)) = 1/(f'(f^{-1}(x)))$
$e = f(0)$, so,
$D_x\left(f^{-1}(0)\right) = 1/\ln e = 1$.

28. D $F'(x) = \frac{d}{dx}\int_0^{g(x)} f(t)\,dt$ and
using the 2nd Fund. Thm of Calc.,
$\frac{d}{dx}\int_0^{g(x)} f'(t)dt = f(g(x))\bullet g'(x)$
So, $F'(1) = f(g(1))g'(1)$

22. A $S_n = \frac{(n+1)!-1}{(n+1)!}$ and $\lim_{n\to\infty} 1 - \frac{1}{(n+1)!} = 1$

29. E Avg. rate = $20(3t_0 - 60)$

$20(3t_0 - 60) = 60 \Rightarrow t_0 = 21$.

23. B $y = x^{\frac{1}{x}}$, $\ln y = \ln x / x$
using L'Hopital's: $(1/x)/1 = 0$
so: $\lim_{x\to\infty} \ln y = 0$ and $\lim_{x\to\infty} y = e^0 = 1$
and y = 1 is a horiz. asymptote.

30. E None are true.

24. C Squaring, $\sin^2 x + 2\sin x \cos x + \cos^2 x$
$= 1 + \sin 2x = 1/25$. So, $\sin 2x = -24/25$,
and $\cos 2x = 7/25$.

25. B Using $2\cos^2 30° - 1 = \cos 60°$ *and* $\sin 60° = 2\sin 30°\cos 30°$,
we get $2\cos^2 30° + 2i\sin 30°\cos 30° = 2(\cos 30°)(cis 30°)$
$= \left(\sqrt{3}\,cis 30°\right)^{12} = 3^6 cis 360°$ *or* $3^6 + 0i$.

26. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$
$\frac{dA}{dt} = \frac{dr}{dt} \Rightarrow 1 = 2\pi r \Rightarrow r = 1/2\pi$
$3 < \pi < 4 \Rightarrow 6 < 2\pi < 8 \Rightarrow 1/8 < 1/2\pi < 1/6$
so: $.1 < r < .2$