Mu Alpha Theta National Convention 2004
Alpha Gemini – no calculators allowed
For each question, NOTA means "none of the above are correct"

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Answers

#	Answer	#	Answer
1	D	18	A
2	A	19	C
3	C	20	D
4	B	21	A
5	C	22	C
6	B	23	B
7	A	24	C
8	B	25	A
9	A	26	D
10	D	27	D
11	C	28	B
12	C	29	A
13	B	30	C
14	C	TB1	$\frac{2}{5}$
15	A	TB2	12
16	D	TB3	
17	C		

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SOLUTIONS

1. Ans: D If you call the total distance 3d for convenience, the time for the first lap is $\frac{d}{x}$ and the time for the second lap is $\frac{2d}{y}$. Average rate = $\frac{\text{total distance}}{\text{total time}}$ = $\frac{3d}{\frac{d}{x}+\frac{2d}{y}} = \frac{3d}{\frac{dy+2dx}{xy}} = \frac{3dxy}{dy+2dx} = \frac{3xy}{2x+y}$

2. Ans: A When an odd number y is raised to any integer power, no factors of 2 are introduced and the result is odd.

3. Ans: C If Jean can do the whole job in 10 hours, then it takes her $\frac{1}{3}\times 10 = 3\frac{1}{3}$ hours to do $\frac{1}{3}$ of the job. If Dan can do the whole job in 12 hours, then it takes him $\frac{2}{3}\times 12 = 8$ hours to do the other $\frac{2}{3}$ of the job. Thus, together, it takes them $3\frac{1}{3}+8 = 11\frac{1}{3}$ hours, or 11 hours and 20 minutes.

4. Ans: B The 15 girls scores add up to 15x. Let the average boys score be y. The 10 boys scores add up to 10y. The 25 students scores average 90 and therefore add up to 25(90) =2,250. 25x + 10y = 2,250 y = $\frac{2{,}250-15x}{10} = 225-\frac{3}{2}x$

5. Ans: C It drops 10 and rebounds 8. It drops 8 and rebounds 6.4, and so forth.
10 + 8 + 8 + 6.4 + 6.4 + 5.12 + 5.12 = 49.04

6. Ans: B The lines intersect when $m_1x+4 = m_2x+3$. $m_2x-m_1x = 1$, $x(m_2-m_1) = 1$, $m_2-m_1 = \frac{1}{x}$, When x is positive, then $m_2 > m_1$

7. Ans: A Since $\lceil x \rceil$ and $\lfloor x \rfloor$ will differ by 1, $\lceil x \rceil - \lfloor x \rfloor = 1$ or $\lceil x \rceil = \lfloor x \rfloor + 1$
$f(x) = \lceil x \rceil + \lfloor x \rfloor = \lfloor x \rfloor + 1 + \lfloor x \rfloor = 2\lfloor x \rfloor + 1$

8. Ans: B

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$k=0 \Rightarrow (-1)^0 2(0)=0;\ \ k=1 \Rightarrow (-1)^1 2(1)=-2;\ \ k=2 \Rightarrow (-1)^2 2(2)=4;\ \ k=3 \Rightarrow (-1)^3 2(3)=-6$

$k=4 \Rightarrow (-1)^4 2(4)=8;\ \ k=5 \Rightarrow (-1)^5 2(5)=-10;\quad 0+-2+4+-6+8+-10=-6$

9. Ans: A Consider the different cases Case 1, when $x \geq 3,\ x+x-3>3,\ 2x>6,\ x>3$
Case 2, when $0<x<3,\ x+(-x)+3>3,\ 3>3$, And it doesn't work. Case 3, when
$x<0,\ (-x)+(-x)+3>3,\ -2x>0,\ x<0$ Therefore, the solution set is
$\{x : x>3 \text{ or } x<0\}$

10. Ans: D Since $(1-2x)^2$ is a squared term, its smallest possible value is zero.
when $x=\frac{1}{2}$

11. Ans: C You are creating a new rectangular solid with dimensions of 5 meters by 3 meters
by x. The new rectangle has volume of 9 times 12 or 108 cubic meters 5(3)(x) = 108;
x = 7.2 The original water was $\frac{1}{2}$ of 25 meters or 12.5 meters deep. The new depth is
12.5 + 7.2 = 19.7.

12. Ans: C
$f(g(x))=g(f(x))$ if they are the same function or if they are inverse functions.
$y=\frac{1}{2}x-4 \quad x=\frac{1}{2}y-4 \quad y=2x+8 \quad f^{-1}(x)=2x+8$ Therefore, II and III satisfy

13. Ans: B $i^{274}=i^{272}(i^2)=1(-1)=-1$

14. Ans: C Using synthetic division

	1	-6	10	42	25
-1	1	-7	17	25	0
-1	1	-8	25	0	

$x^2-8x+25=0$
$x=\frac{8\pm\sqrt{64-4(1)(25)}}{2}=\frac{8\pm\sqrt{-36}}{2}$
$\frac{8\pm 6i}{2}=4\pm 3i$

15. Ans: A
$\frac{6x^3-31x^2-15x+56}{x^3(x-8)}=\frac{A}{x-8}+\frac{B}{x^3}+\frac{C}{x^2}+\frac{D}{x}$
$Ax^3+B(x-8)+Cx(x-8)+Dx^2(x-8)=6x^3-31x^2-15x+56$

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If $x = 0$, $-8B = 56$ $B = -7$ If $x = 8$, $A(8)^3 = 6(8)^3 - 31(8)^2 - 15(8) + 56$ $A = 2$

$(A + D)x^3 = 6x^3$ $D = 4$ $(B - 8C)x = -15x$ $-7 - 8C = -15$ $C = 1$ $\frac{2}{x-8} - \frac{7}{x^3} + \frac{1}{x^2} + \frac{4}{x}$

16. Ans: D Satisfying intersection points are at (0,4), (4,0), (6,4), (3,7). Using the points in order z = 3(0) + 4(4) = 16, z = 3(4) + 4(0), z = 3(6) + 4(4), z = 3(3) + 4(7) Maximum value of 37 at (3,7)

17. Ans: C 50% have taken Statistics and of those, 70% did not get an A in Calculus. $(.5)(.7) = .35$

18. Ans: A $1 - \sin^2 x - \sin x + 1 - 0$; $\sin^2 x + \sin x - 2 = 0$; $(\sin x + 2)(\sin x - 1) = 0$; $\sin x = -2$ or $\sin x = 1$; $x = \frac{\pi}{2}$

19. Ans: C $(-3, -240^\circ)$ is the same as $(3, -60^\circ)$. This creates a 30, 60, 90 triangle with hypotenuse of 3. The x coordinate is $\frac{3}{2}$ and the y coordinate is $\frac{-3\sqrt{3}}{2}$.

20. Ans: D This is the trig equivalent of $\sin\left(\frac{7\pi}{24} - \frac{\pi}{8}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

21. Ans: A

$\cos(\arcsin x - \arctan x) = \cos(\arcsin x)\cos(\arctan x) + \sin(\arcsin x)\sin(\arctan x) =$

$\frac{\sqrt{1-x^2}}{1}\left(\frac{1}{\sqrt{1+x^2}}\right) + \frac{x}{1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \frac{\sqrt{1-x^2}+x^2}{\sqrt{1+x^2}}$

22. Ans: C The center is the midpoint of the vertex points (2,3). The conjugate axis is 8 units long and the transverse axis is 6 units long and parallel to the x axis. The equation is $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1$

23. Ans: B The angle is co terminal with $\frac{\pi}{2}$ and the -2 puts the point at $(0.-2)$.

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24. Ans: C With substitutions the equation becomes

$x^2 + y^2 = \dfrac{x}{\sqrt{x^2 + y^2}}$ or $\left(x^2 + y^2\right)^{\frac{3}{2}} = x$ or

$\left(x^2 + y^2\right)^3 = x^2$

25. Ans: A $r + 2r\cos\theta = 2$; $\sqrt{x^2 + y^2} + 2x = 2$; $x^2 + y^2 = 4 - 8x + 4x^2$; $y^2 - 3x^2 + 8x = 4$

$y^2 - 3(x^2 - \frac{8}{3}x + \frac{16}{9}) = 4 - \frac{16}{3}$; $y^2 - 3\left(x - \frac{4}{3}\right)^2 = \frac{-4}{3}$; $\dfrac{\left(x - \frac{4}{3}\right)^2}{\frac{4}{9}} - \dfrac{y^2}{\frac{4}{3}} = 1$

conjugate axis$= 2\left(\sqrt{\frac{4}{3}}\right) = \dfrac{4\sqrt{3}}{3}$

26. Ans: D $\ln\left(4 - x^2\right) \geq 0 \Rightarrow \left(4 - x^2\right) \geq 1 \Rightarrow -x^2 \geq -3 \Rightarrow x^2 \leq 3 \Rightarrow -\sqrt{3} \leq x \leq \sqrt{3}$

27. Ans: D This is the derivative by the definition of
$f\left(x\right) = 3x^4 - 5x^2 + 6$ evaluated at $x = 2$.

$f'\left(x\right) = 12x^3 - 10x$ evaluated at $x = 2$ gives 76. It also equals

$$\lim_{\Delta x \to 0} \frac{28 + 28\Delta x + 7\Delta x^2 + 48\Delta x + 48\Delta x^2 + 12\Delta x^3 + 12\Delta x^2 + 12\Delta x^3 + 3\Delta x^4 - 28}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \frac{\Delta x\left(76 + 67\Delta x + 24\Delta x^2 + 3\Delta x^3\right)}{\Delta x} = 76$$

28. Ans: B Discontinuities occur at

$x = -2$ $\left(\lim_{x \to -2^-} f\left(x\right) = -2 \text{ while } \lim_{x \to -2^+} f\left(x\right) = 2\right)$ and at $x = \dfrac{\pi}{2}$ and $x = \dfrac{3\pi}{2}$

$\left(\tan x \text{ is undefined at these values}\right)$. The number of points of discontinuity is 3.

29. Ans: A When

$x = 3$, $2\left(3\right) = t + \sqrt{t^2 - 1} \Rightarrow 6 - t = \sqrt{t^2 - 1} \Rightarrow t^2 - 12t + 36 = t^2 - 1 \Rightarrow t = \dfrac{37}{12}$

Now evaluate y using the first equation: $y = 3\left(\frac{37}{12}\right) - 3\sqrt{\left(\frac{37}{12}\right)^2 - 1} \Rightarrow y = \frac{37}{4} - 3\left(\frac{35}{12}\right) = \frac{37}{4} - \frac{35}{4} = \frac{1}{2}$

30. Ans: C

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$\frac{2x^3+9x^2+7x-6}{x^2-2x-3} = 2x+13+\frac{39x+33}{x^2-2x-3}$ so the oblique asymptote has equation $y = 2x = 13$.

the area is a trapezoid, and $A = \frac{1}{2}(2)(15+19) = 34$.

Tiebreaker 1: Ans: $\frac{2}{5}$ As $x \to +\infty$, both $\frac{1}{3x}$ and $\frac{1}{6x}$ approach 0. $\lim_{x\to+\infty} \frac{2x}{5x} = \frac{2}{5}$.

Tiebreaker 2: Ans: 12 The total points for males were 8(87) = 696. If x females took the test, the total points for females would be 92x. The average for all students would be

$\frac{696+92x}{8+x} = 90 \Rightarrow 696+92x = 720+90x \Rightarrow 2x = 24 \Rightarrow x = 12$