

ALPHA GEMINI TEST - ALPHA LEVEL
2000 Mu Alpha Theta National Convention

1. What is the period of the graph of the equation $y = 4 \tan 3x$?
A. $\frac{\pi}{4}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $\frac{2\pi}{3}$ E. NOTA
2. T. J. drives from San Diego to Geminopolis at an average speed of 60 mph, and from Geminopolis to San Diego at an average speed of 50 mph. What is T. J.'s average speed for the entire trip?
A. 54 mph B. $54\overline{54}$ mph C. 55 mph D. $55\overline{45}$ mph E. NOTA
3. A rope is attached on one end to a point on a unit sphere. What should the length of the rope be so that the other end may reach 50% of the points on the sphere?
A. $\frac{\pi}{4}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. π E. NOTA
4. A digital calendar displays the month as numbers 1 through 12, date from 1 to 31, and time on a 12-hour system. For example, February 27 at 4 o'clock p.m. would read "2-27 4:00" on the calendar. For how many minutes during the year 2001 will the calendar show at least one digit which is a 6?
A. 134748 B. 150516 C. 166824 D. 191400 E. NOTA
5. $4^x + 4^{2x} + 4^x =$
A. 4^{4x} B. $4^{x^2} + (2)4^x$ C. $4^{2x} + 2^{4x}$ D. $16^x + 4^{x+\frac{1}{2}}$ E. NOTA
6. What is the sum of all values of m which will make one solution to $2x^3 + mx^2 + mx + 5 = 0$ an integer?
A. $-\frac{1}{2}$ B. $-\frac{1}{4}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA
7. How many distinct real solutions exist for the equation $(x^2 + 6x + 4)^{x^2+6x+10} = 16$?
A. 2 B. 4 C. 6 D. 8 E. NOTA
8. Evaluate: $\lim_{x \rightarrow n^2} \frac{\sqrt{x} - n}{x - n^2}$, where n is positive.
A. $\frac{1}{2n}$ B. $\frac{1}{2\sqrt{n}}$ C. $\frac{1}{n^2}$ D. $\frac{1}{2n^2}$ E. NOTA
9. The integers 2467 and 1378 have the following properties:
(1) The digits are in ascending order
(2) All of the digits are different
How many more integers between 1000 and 9999 have these properties?
A. 120 B. 122 C. 124 D. 126 E. NOTA

ALPHA GEMINI TEST - ALPHA LEVEL
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Page 2

10. If $x + y = a$ and $xy = b$, then $x^3 + y^3$ in terms of a and b is
- A. $\frac{a^2}{3b}$ B. $a^3 - 3ab$ C. $\frac{a^3}{3b}$ D. $a^3 + 3ab$ E. NOTA
11. What is the largest integer value of k so that 7^k evenly divides $400!$?
- A. 57 B. 58 C. 65 D. 66 E. NOTA
12. Find the product of x and y if $y = 4 \log_4 x + 1$ and $y - 3 = 2 \log_4 x$.
- A. 20 B. 24 C. 36 D. 40 E. NOTA
13. A boat begins from a starting point, with the boat's bow pointing due north at all times and the boat's engine pushing the boat due north traveling 30 miles per hour at all times. After 40 minutes of calm sailing, a wind blowing from the east at 15 mph alters the boat's course for 30 minutes. There is no wind for the next hour. Then a 60 mph wind from the west to the east changes the course for 10 minutes. To the nearest tenth of a mile, how far is the boat now from its starting point?
- A. 70.0 B. 73.9 C. 130.0 D. 133.9 E. NOTA
14. $.\overline{12}_3 + .\overline{45}_6 = \frac{M}{N}$, a fraction in base 10 such that $\text{GCF}(M, N) = 1$. What is $M + N$?
- A. 596 B. 626 C. 657 D. 687 E. NOTA
15. If $p + q + r + s = 10$, and $p, q, r,$ and s are all natural numbers (but not necessarily distinct), how many different ordered 4-tuples (p, q, r, s) satisfy the equation?
- A. 84 B. 165 C. 210 D. 220 E. NOTA
16. Three players, Andrea, Bonnie, and Cathy, play a game. Each takes turns rolling a fair four-sided die with the possible outcomes being a 1, 2, 3, or 4. The first to roll a 4 wins. What is the probability that Bonnie wins if she plays second?
- A. $\frac{11}{37}$ B. $\frac{12}{37}$ C. $\frac{1}{3}$ D. $\frac{13}{36}$ E. NOTA
17. What is the thousands digit of 5^{2000} plus the tens digit of 7^{2000} ?
- A. 3 B. 7 C. 8 D. 12 E. NOTA
18. What is the missing term of this sequence: 2, 3, 5, 7, 13, 15, 21, 23, 27, ___?
- A. 31 B. 33 C. 35 D. 37 E. NOTA

ALPHA GEMINI TEST - ALPHA LEVEL

2000 Mu Alpha Theta National Convention

Page 4

26. Given $\triangle JKL$, where $m\angle J = 41^\circ$, $JK = 10$, and $KL = 7$. Which of the following could be the product of the length of side JL and the two degree measures of angles K and L ? Round only the final answer to the nearest ten.
- A. 16120 B. 24200 C. 37920 D. 48210 E. NOTA
27. What is the radius of a circle which has the same area as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?
- A. ab B. $(ab)^2$ C. $a+b$ D. \sqrt{ab} E. NOTA
28. Evaluate $\prod_{n=1}^{123456} (i^{3n+7})$, where $i = \sqrt{-1}$.
- A. 1 B. -1 C. i D. $-i$ E. NOTA
29. Which of the following is equivalent to the logical expression $[(p \vee q) \leftrightarrow (\sim q \rightarrow \sim p)] \rightarrow [(q \rightarrow \sim p) \wedge q]$?
- A. $p \wedge q$ B. $p \vee q$ C. $\sim(p \wedge q)$ D. $\sim(p \vee q)$ E. NOTA
30. The sum of three positive integers is 78, and their product is 16530. What is the largest of these integers?
- A. 29 B. 30 C. 38 D. 57 E. NOTA