2000 Mu Alpha Theta National Convention

"NOTA" in each question denotes "None of the Above"

1) If f is a linear function, $f(3)=14$ and $f(-6)=-16$, then $f(0)=?$

A) –10 B) 0 C) 4 D) 12 E) NOTA

2) Which of the following functions have the property: $f(x)=f^{-1}(x)$?

I) $f(x)=x$ II) $g(x)=\frac{4}{x}$ III) $h(x)=\frac{x}{x-1}$ IV) $i(x)=\frac{x-2}{x}$

A) I, II, III, and IV B) I, II, and III only C) I and II only D) I only E) NOTA

3) What is the range of $y(x)=\frac{3x^2}{(3-x)(x-2)}$?

A) $(-\infty,-3)\cup(-3,0]$ B) $(-\infty,0]$ C) $(-\infty,-3)\cup(-3,0)$ D) $\Re$ E) NOTA

4) If $f(x)=\lceil x \rceil$ denotes the least integer which does not precede x, find the sum of the values of $f(x)$ for which $x^2<\pi^4$.

A) –10 B) –9 C) 9 D) 10 E) NOTA

5) $P(x)$ denotes an odd function such that $P(1)=2, P(3)=5$, and $P(-5)=-1$.
What is the value of $P[P(P(-3))]$?

A) –5 B) –2 C) 2 D) 5 E) NOTA

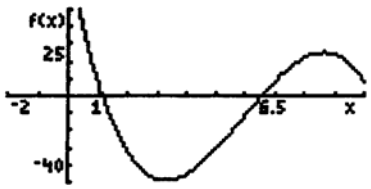
6) A region is bounded above by $y=1-x^2$ and below by $y=x^2-1$. Find the area of the largest rectangle that can be circumcribed by these 2 functions.

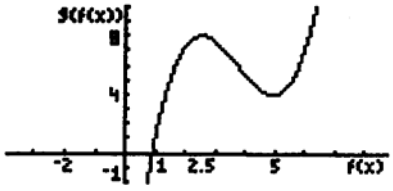
A) $\frac{4\sqrt{3}}{9}$ B) 1 C) $\frac{8\sqrt{3}}{9}$ D) 2 E) NOTA

7) $f(x)=\begin{cases}2, x=0\\ \frac{1}{f(x-2)}, x\neq 0\end{cases}$ Find $f(100)-f(-50)$.

A) –2 B) $-\frac{3}{2}$ C) 0 D) $\frac{3}{2}$ E) NOTA

8) Let functions f and g be represented by the following graphs:





Given the above information, at how many x-values does the relative minimum of g occur on $\{0 \le x \le 9\}$?

A) Zero B) One C) Two D) Three E) NOTA

9) Find an expression for $f(4x)$ in terms of $f(x)$, given $f(x) = \frac{x}{x-1}$.

A) $\frac{4f(x)}{3f(x)-1}$ B) $\frac{4f(x)}{3f(x)+1}$ C) $\frac{4f(x)}{4f(x)-1}$ D) $\frac{4f(x)}{4f(x)+1}$ E) NOTA

10) If a random point lies within the ellipse defined by $x^2 + y^2 = 36$, the probability of a random point falling within the smaller region bounded by the function $f(x) = -\frac{1}{2}|x| + 3$ and the ellipse is closest to:

A) $\frac{1}{3}$ B) $\frac{27}{80}$ C) $\frac{31}{90}$ D) $\frac{384}{1111}$ E) NOTA

11) Jennifer, Kellie, Leslie, and Stephanie are all trying to determine the equation of a parabolic function, given that it passes through the points $(-2,35), (1,-10)$, and $(4,71)$. Each receives the following as the sum of the coefficients of the function, expressed as a quadratic polynomial:
Jennifer: -9 Kellie: -8 Leslie: -7 Stephanie: -6
Which of the girls could possibly be correct?

A) Jennifer B) Kellie C) Leslie D) Stephanie E) NOTA

12) The domain of the function $f(x) = \sin\left|\sqrt{\ln x + 3}\right|$ is:

A) $x \ge -3$ B) $x \ge 0$ C) $x \ge e^{-3}$ D) $x \ge 1$ E) NOTA

13) Given $f(3)=4, f(5)=8$, and $f(x)=\dfrac{f(x-2)}{f(x-1)}$, find $f(0)$.

A) 8 B) 16 C) 32 D) 64 E) NOTA

14) **Let A = the sum of the rational roots and B= the product of the irrational roots of** $f(x)=\ln(4x^4-8x^3-17x^2+40x-14)$. **Find A+B.**

A) –5 B) –4 C) –3 D) –2 E) NOTA

15) Find all of the linear asymptotes to the graph of $x^3-x^2y-3xy-1=0$.

A) $x=0, x=-3$
B) $x=1,\ y=x$
C) $x=0, x=-3,\ y=x-3$
D) $x=0, x=1, x=-3$
E) NOTA

16) How many of the following functions are either even but not odd, or odd but not even?
I) $y_1=x^2-x$ II) $y_2=\cos(\tan x)$ III) $y_3=(x-1)^3+3(x-1)^2+3(x-1)^1+(x-1)^0$ IV) $y_4=0$

A) One B) Two C) Three D) Four E) NOTA

17) Let $\beth$ denote the modulus operation such that $\beth(x, y)\equiv x \bmod y$.
Find $\beth(\beth(c,a), \beth(b,d))$ if a = 11, b = -3, c = 40, and d = 7.

A) 1 B) 2 C) 3 D) 4 E) NOTA

18) Find the thousandths digit of the distance between the functions:
$3x+2y=7$ AND $3x+2y=13$

A) 1 B) 2 C) 3 D) 4 E) NOTA

19) Find the area enclosed by $m(x)=\sqrt{10-\dfrac{(x-5)^2}{5}}$ and $w(x)=-\sqrt{10-\dfrac{(x-5)^2}{5}}$,
to the nearest thousandth.

A) 70.248 B) 70.249 C) 70.250 D) 70.251 E) NOTA

20) Which of the following is an onto function?

A) $p(x)=\sin x$ B) $q(x)=\cos x$ C) $r(x)=\tan x$ D) $s(x)=\sqrt{x}$ E) NOTA

21) For a point P in the plane, $G(P)$ is the line through P whose slope is equal to the x-coordinate of P. For 2 non-parallel lines I_1 and I_2 in the plane, $H(I_1,I_2)$ is the intersection point of I_1 and I_2. Given points Q_i, with $Q_i=(i,0)$, find $H\{G[H(G(Q_2),G(Q_{-1}))],G[H(G(Q_1),G(Q_{-2}))]\}$.

A) (-3,0) B) (0,-2) C) (2,0) D) (0,-3) E) NOTA

22) A function $\mathbf{2}$ is defined by $\mathbf{2}(x)$ equals the number of occurrences of the digit 2, when the numbers 0 through x are written. Example: $\mathbf{2}(12)=2$ since 0,1,2,3,... ,11,12 has the digit 2 occurring twice. Find $\mathbf{2}(200)$.

A) 38 B) 39 C) 40 D) 41 E) NOTA

23) Let $k(x)=\sin x$, and let $l(x)=\tan x$. If $k(\cos^{-1}\frac{1}{2})+l(\cos^{-1}\frac{1}{3})$ is reduced to the simplified radical form: $\frac{\sqrt{a}}{b}+c\sqrt{d}$, where $a>d$, find $a+b+c+d$.

A) 9 B) 11 C) 13 D) 14 E) NOTA

24) How many of the following are true, given that $f(x)=\ln x$?

I) $f(ax)=a\cdot f(x)$ II) $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ III) $f(x^a)=f(x)^{f(a)}$ IV) $f^{-1}(1)=0$

A) One B) Two C) Three D) Four E) NOTA

25) The domain of the function $g(x)=\frac{1}{\sqrt{a+x}}+\frac{1}{\sqrt{a-x}}$, given $a<0$, is:

A) $x\geq a$ or $x\leq -a$ B) $x>a$ or $x<-a$ C) $x=a$ D) $a<x<-a$ E) NOTA

26) Find the area inside the region bounded by the polar graph of $r^2 = 100\cos^2\theta$ but outside the region bounded by the polar graphs of $r = \frac{10\cos\theta \pm \sqrt{100\cos^2\theta - 96}}{2}$ and $r = \frac{-10\cos\theta \pm \sqrt{100\cos^2\theta - 96}}{2}$.

A) 48π B) 49π C) 98π D) 99π E) NOTA

27) Let a and b be integers, and suppose $Z(a,b)$ is defined recursively by:

$$Z(a,b) = \begin{cases} 5, a < b \\ Z(a-b, b+2) + a, a \geq b \end{cases}$$

Find Z(15,2).

A) 5 B) 15 C) 27 D) 42 E) NOTA

28) Which of the following is TRUE concerning functions?

A) An invertible function's inverse may not necessarily be a function.
B) An onto and one-to-one function has established a one-to-one correspondence.
C) A one-to-one function has exactly one intersection point with any horizontal line drawn.
D) Not all functions are relations.
E) NOTA

29) If $f'(x)$ is equal to the limit as h approaches 0 of $\frac{f(x+h) - f(x)}{h}$, find $f'(4)$ if $f(x) = x^3 + 1$.

A) 48 B) 49 C) 64 D) 65 E) NOTA

30) Given a function $c(x)$ satisfying $c(x) + 2 \cdot c\left(\frac{1}{1-x}\right) = x$, find $c(2)$.

A) $\frac{2}{3}$ B) $\frac{10}{7}$ C) $\frac{37}{8}$ D) 5 E) NOTA