

1992 Mu Alpha Theta National Convention
Functions Topic Test

All numbers, and the domains and ranges of all functions, are real unless otherwise indicated. The expression $[x]$ represents the greatest integer function of x ; if there is no base given for a logarithm, assume it is the natural log.

1. Find $f^{-1}(8)$ if $f(x) = x^{4/3} - 8$.

- A. 8 B. 4 C. 2 D. $16/3$
E. None of the above

2. If $f(x)$ and $g(x)$ are inverse functions, which of the following is true?

- A. $f(x)g(x) = 1$ B. $f(x) = g(1/x)$ C. $f(g(x)) = x$
D. $f(x)/g(x) = 1$ E. None of the above

3. How many of the following are functions?

- i. $f(x) = |x|$
ii. $f(x) = \sin^{-1}(x)$
iii. $f(x) = [x]$
iv. $f(x) = \log x, x > 0$

- A. None B. One C. Two D. Three E. Four

4. The domain of the function

$$f(x) = \sin(\sqrt{|\log(x) + 3|})$$

is

- A. $x > 0$ B. $x > -3$ C. $x > e^{-3}$ D. $x > 1$
E. None of the above

5. Given that $f(ax) = af(x)$ for all real a , and $f(2) = 5$, find $f(17)$.

- A. 51 B. 34 C. $85/2$ D. $17/2$
E. None of the above

6. Find an expression for $f(4x)$ in terms of $f(x)$, given that $f(x) = x/(x-1)$.

- A. $4f(x)/[3f(x)-1]$ B. $4f(x)/[3f(x)+1]$ C. $4f(x)/[4f(x) - 1]$
D. $4f(x)/[4f(x)+1]$ E. None of the above

7. The roots of $f(x) = x^3 - 12x^2 + 37x - b$ form an arithmetic progression when b is

- A. 20 B. 40 C. 60 D. 80
E. None of the above

8. The domain of the function

$$f(x) = \frac{1}{\sqrt{a+x}} + \frac{1}{\sqrt{a-x}}$$

is

- A. $-a < x < a$ B. $x > a$ or $x < -a$ C. $x > a$
D. The null set E. None of the above

9. How many of the following functions are continuous?

- i. $f(x) = 1/x$
ii. $f(x) = |x|$
iii. $f(x) = [x]\sin(\pi x)$
iv. $f(x) = e^{|x|}$

- A. None B. One C. Two D. Three E. Four

10. Find the sum of the rational roots of $f(x) = \ln(4x^4 - 8x^3 - 17x^2 + 40x - 14)$.

- A. $-5/2$ B. -2 C. 2 D. $5/2$
E. None of the above

11. Given $f(ax) = \log_a x$, find $f(x)$.

- A. $(\log_a x)/a$ B. $a \log_a x$ C. $(\log_a x)/a + 1$
D. $\log_a x - 1$ E. None of the above

12. If $f(x) = (ax+b)/(cx+d)$, find $f^{-1}(x)$.

- A. $(cx-d)/(ax-b)$ B. $(dx-b)/(-cx+a)$
C. $((x/a)-b)/((x/c)-d)$ D. $((x/c)-b)/(-(x/a)+d)$
E. None of the above

13. A polynomial of odd degree with real coefficients can have

- A. all nonreal roots B. an odd number of real roots
C. an odd number of nonreal roots D. an even number of roots
E. None of the above

14. How many of the following functions have a well-defined inverse function?

i. $f(x) = e^x$

ii. $f(x) = \frac{e^x + e^{-x}}{2}$

iii. $f(x) = \frac{e^x - e^{-x}}{2}$

iv. $f(x) = e^{x^2}$

A. None B. One C. Two D. Three E. Four

15. Find the sum of the squares of all roots (real and nonreal) of

$$f(x) = x^3 + 6x^2 - 12x + 10.$$

A. 15 B. 30 C. 45 D. 60
E. None of the above

16. The domain of a function $f(n)$ is the positive integers between 1 and 17 inclusive. Given that $f(1) = 1$ and for $n > 1$, $f(n) = f(n-1) + f(n-2) + \dots + f(1)$, find the sum of all elements in the *range* of $f(n)$.

A. $2^8 - 1$ B. $2^9 - 1$ C. $2^{16} - 1$ D. $2^{17} - 1$
E. None of the above

17. How many of the following are true, given that $f(x) = \log x$?

i. $f(ax) = af(x)$

ii. $f(x/y) = f(x) - f(y)$

iii. $f(x^a) = f(x)^{f(a)}$

iv. $f^{-1}(1) = 0$

A. None B. One C. Two D. Three E. Four

18. Given a function $f(n)$ on the positive integers, with $f(p) = p^{1/2}$ for every prime p and $f(ab) = f(a)f(b)$ for integers a and b , how many of the following are true?

i. $f(1) = 1$

ii. $f(p^k) = p^k$ for primes p

iii. $f(12)$ is real

iv. $f(a+b) = f(a) + f(b)$ for integers a and b

A. None B. One C. Two D. Three E. Four

19. How many of the following sets of functions have the property that, given any two elements $f(x)$ and $g(x)$ of the set, the composition $g(f(x))$ is in the set?

- i. functions of the form $ax+b$
- ii. functions of the form ax^2+bx+c
- iii. polynomial functions
- iv. polynomial functions with 12 as a root

A. None B. One C. Two D. Three E. Four

20. The function $f(x,y,a) = x^2 - y^2 - a$ has zeroes with $|y| \geq |x|$ if and only if

- A. $a < 0$ B. $a = 0$ C. $a > 0$ D. More than one of A, B, and C
E. None of the above

21. A function is said to be *onto* its range if every element of the range is the function of some element of the domain. How many of the following functions from the interval $[0,1]$ to the interval $[-5,5]$ are onto?

- i. $f(x) = \sin 10\pi x$
- ii. $f(x) = 5 \sin 2\pi x$
- iii. $f(x) = 10x - 5$
- iv. $f(x) = 6x^2 + 4x - 5$

A. None B. One C. Two D. Three E. Four

22. Given the function $F(x,y)$ such that $F(x,y) = x - y$ for $0 \leq x,y < 1$, and $F(x+1,y) = F(x,y+1) = F(x,y)+1$, find $F(25/4,16/3)$.

- A. $131/12$ B. $133/12$ C. $-131/12$ D. $-133/12$
E. None of the above

23. Given $f(x)$ such that $f(1-x) + (1-x)f(x) = 5$, find $f(5)$.

- A. $21/19$ B. 5 C. -4 D. $-20/21$
E. None of the above

24. Given $f(x) = ae^{bx}$ and $g(x) = c \ln(dx)$, with none of a, b, c, d equal to zero, $f(g(x)) = g(f(x))$ for all x

- A. Never B. If and only if $bc = ad$
C. If and only if $ad = 1$ D. If and only if $bc = ad = 1$
E. None of the above

25. Call a function $F(x,y)$ from the real plane to itself a *distance function* if (1) $F(x,y) \geq 0$ for all x and y , (2) $F(x,y) = 0$ if and only if $x = y = 0$, and (3) $F(x,y) \leq F(x,0) + F(0,y)$ for all x and y . How many of the following are valid distance functions?

- i. $F(x,y) = |x| + |y|$
 - ii. $F(x,y) = x^3 + y^3$
 - iii. $F(x,y) = e^x$
 - iv. $F(x,y) = (x^2 + y^4)^{1/2}$
- A. None B. One C. Two D. Three E. Four

26. Consider a family of functions $f_b(x)$ such that (1) $f_b(0) = b$ and (2) for any a , $f_b(x) = 2^a f_b(x-a)$. Find an expression for $f_c(2x)$ in terms of $f_b(x)$.

- A. $(c/b)f_b(x)$
- B. $e^{c-2b}(f_b(x))^2$
- C. $(c/b^2)[f_b(x)]^2$
- D. $(c-b) + (c/b)f_b(x)$
- E. None of the above

27. Given A , a set of 13 (distinct) elements. What fraction of the functions from A to A have a well-defined inverse function?

- A. $1/13$
- B. $13!/13^{12}$
- C. $12!/13^{12}$
- D. $12!/13^{13}$
- E. None of the above

28. Given a function $f(x)$ satisfying $f(x) + 2f(1/(1-x)) = x$, find $f(2)$.

- A. 5
- B. $2/3$
- C. $37/8$
- D. $10/7$
- E. None of the above

29. The function $F(x,y)$ takes a point (x,y) in the real plane to the point $(ax + by + e, cx + dy + f)$. Find a restriction on the coefficients a through f so that the distance between the images under F of two points is the same as the distance between the two points.

- A. $a + b + e = c + d + f$
- B. $ab + e = cd + f$ and $ac = bd$
- C. $a^2 + c^2 = b^2 + d^2 = 1$ and $ab + cd = 0$
- D. $abc + bcd + abd + acd = 1$
- E. None of the above

30. For a point P in the plane, $G(P)$ is the line through P whose slope is equal to the x -coordinate of P . For two non parallel lines in the plane l_1 and l_2 , $H(l_1, l_2)$ is the intersection point of l_1 and l_2 . Given points Q_i , with $Q_i = (i, 0)$, find

$$H\{G[H(G(Q_2), G(Q_1))], G[H(G(Q_1), G(Q_2))]\}$$

- A. $(-3, 0)$
- B. $(0, -2)$
- C. $(2, 0)$
- D. $(0, -3)$
- E. None of the above