

**Functions Topic Test Key**

1. B
2. C
3. C
4. D
5. C
6. C
7. A
8. B
9. E
10. B
11. D
12. E
13. D
14. A
15. B
16. C
17. E
18. C
19. D
20. C
21. C
22. C
23. C
24. B
25. B

# Functions Topic Test

## Solutions

5

①  $f(x) = x^3 + 2x^2 - 5$

ans B

$$\begin{aligned} f(2) + f(-1) &= 2^3 + 2^2 - 5 + (-1)^3 + 2 - 5 \\ &= 11 - 1 - 3 \\ &= 7 \end{aligned}$$

②  $f(x) = \log_b x$

ans C

$$\frac{-3}{2} = \log_b \frac{1}{8}$$

$$b^{-\frac{3}{2}} = 2^{-3}$$

$$b^{\frac{1}{2}} = 2$$

$$b = 4$$

ans C

③  $f(x) = \frac{x}{x+1}$        $(x, 3)$

$$3 = \frac{x}{x+1}$$

$$3x+3 = x$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

ans D

④  $f(x) = 4^x$

$$\begin{aligned} f(x+2) - f(x+1) + f(x) &= 4^{x+2} - 4^{x+1} + 4^x \\ &= 16 \cdot 4^x - 4 \cdot 4^x + 4^x \\ &= 13 \cdot 4^x \end{aligned}$$

ans C

⑤  $\begin{cases} 3 = 16 + 4A + 2B - 5 \\ -37 = -16 + 4A - 2B - 5 \end{cases}$

$$\begin{aligned} -34 &= 8A - 10 & 3 &= 11 - 12 + 2B \end{aligned}$$

$$8A = -24$$

$$2B = 4$$

$$A = -3$$

$$B = 2$$

$$A + B = -1$$

$$\textcircled{6} \quad f(x+1) = x^2 + 3x + 5$$

$$f(x) = ax^2 + bx + c$$

$$x^2 + 3x + 5 = a(x+1)^2 + b(x+1) + c$$

$$x^2 + 3x + 5 = ax^2 + 2ax + a + bx + b + c$$

$$a = 1$$

$$2 + b = 3 \therefore b = 1$$

$$a + b + c = 5$$

$$c = 3$$

$$\therefore f(x) = x^2 + x + 3$$

alt. sol

decrease roots by 1

$$\begin{array}{r|rrr} -1 & 1 & 3 & 5 \\ & & -1 & -2 \\ \hline & 1 & 2 & 3 \end{array}$$

$$f(x) = x^2 + x + 3$$

Ans C

$$\textcircled{7} \quad \frac{4}{2x-1} = 2 \left( \frac{4}{x-1} \right)$$

$$\frac{1}{2x-1} = \frac{2}{x-1}$$

$$x-1 = 4x-2$$

$$-3x = -1$$

$$x = \frac{1}{3}$$

Ans A

$$\textcircled{8} \quad P(x) = (x+2)(x+k)$$

$$P(x) = x^2 + (2+k)x + 2k$$

$$\begin{array}{r|rrr} 1 & 1 & 2+k & 2k \\ & & 1 & 3+k \\ \hline & 1 & 3+k & 3+3k \end{array}$$

$$3 + 3k = 12$$

$$k = 3$$

alt sol:

$$12 = 3(1+k)$$

$$k = 3$$

Ans B

⑨  $y = x^2 - 3x + 2$   
 $y = x^2 - 3x + \frac{9}{4} + 2 - \frac{9}{4}$       Ans. E  
 $y = x^2 - 3x + \frac{9}{4} - \frac{1}{4}$   
 $y + \frac{1}{4} = (x - \frac{3}{2})^2 - \frac{1}{4}$   
 $\therefore y \geq -\frac{1}{4}$

⑩  $f(x) = \ln x$        $g(8) = e^8$       Ans B  
 $g(x) = e^x$   
 $f(e^8) = \ln e^8$   
 $f(e^8) = 8 \ln e$   
 $= 8$

⑪ Must be either C or D      Ans. D

(4 has to be a factor of last no.)

C)  $(x^2 - 4)(x^2 - 1)$  No

$\therefore$  Must be D. Check.

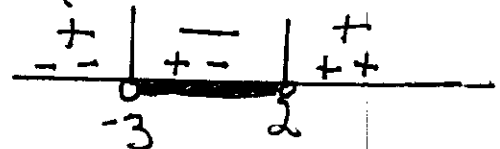
2	1	-6	9	4	-12	
		2	-8	2	+12	
2	1	-4	1	6	0	
		2	-4	-6	0	yes
	1	-2	-3	0		

⑫  $D_{g \circ f}$ . Take values in  $D_f$  such that  $f(x)$  is in  $D_g$ .      Ans. E

$\therefore x^2 + x - 1 < 5$

$x^2 + x - 6 < 0$

$(x+3)(x-2) < 0$



Take  $(-3 < x < 2) \cap (x \geq -2)$

$-2 \leq x < 2$

$$(3) (h \circ g \circ f)(x) = h[g(f)(x)]$$

$$g[f(x)] = \frac{1}{cx+d}$$

$$h[g(f(x))] = \frac{a}{c} + \left(\frac{bc-ad}{c}\right) \frac{1}{cx+d}$$

$$= \frac{acx + ad + bc - ad}{c(cx+d)}$$

ans D

$$= \frac{c(ax+b)}{c(cx+d)}$$

$$(14) y = \frac{4x^3 + 4x^2 - 3x - 3}{x^2 + 3x + 2}$$

ans A

$$y = \frac{4x^2(x+1) - 3(x+1)}{(x+1)(x+2)}$$

$$y = \frac{\cancel{(x+1)}(4x^2 - 3)}{\cancel{(x+1)}(x+2)}$$

$$x+2 \overline{) 4x^3 + 4x^2 - 3x - 3}$$

$$\underline{4x^2 + 8x \phantom{- 3}}$$

$$-8x - 16$$

$$\underline{-8x - 16}$$

$$19$$

Asymptotes  $x = -2$  or  $y = 4x - 8$

$$(15) \lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}$$

ans B

$$\lim_{x \rightarrow 2} \frac{2x+5 - x-7}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})}$$

$$\lim_{x \rightarrow 2} \frac{\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{2x+5} + \sqrt{x+7})} = \frac{1}{3+3}$$

$$= \frac{1}{6}$$

⑩  $f \circ g(x) = x \rightarrow f$  and  $g$  inverses  
 $\therefore$  Find  $f^{-1}(x)$  which will be  $g(x)$  Ans C

$$x = \frac{y+1}{y}$$

$$xy = y+1 \quad \therefore g(x) = \frac{1}{x-1}$$

$$y = \frac{1}{x-1} \quad g\left(\frac{x-1}{x}\right) = \frac{1}{\frac{x-1}{x}-1} = \frac{x}{-1}$$

⑪

2	1	-6	5	12	E
		2	-8	-6	
2	1	-4	-3	6	
		2	-4		
2	1	-2	-7		
		2			

1    0

$$x^3 - 7x + 6 = 0$$

you could also use sum of roots:

$$r_1 + r_2 + r_3 = 6$$

if roots 2 less, sum will be 0.

⑫ Just possible fractions    Ans C  
 $\pm \frac{2}{3}, \pm \frac{1}{9}, -\frac{1}{3}$     D & E can't be.

$-\frac{1}{9}$	9	19	65	115	66	6	✓
		-1	-2	-7	-12	-6	
	9	18	63	108	54	0	

$$(19) \quad 16 + 8r_1 = 0 \rightarrow r_1 = -2$$

$$200 - 40 - r_2 = 0 \rightarrow r_2 = 160$$

$$r_1 + r_2 + 10 = 2$$

$$r_1 = -4$$

$$32 + 16 - r_3 = 0 \rightarrow r_3 = 48$$

$$r_1 + r_2 + r_3 = 206$$

ans D

(20) A and B can be.

check C

$$y = e^x \quad f^{-1}: x = e^y$$

$$\ln x \neq e^x \quad y = \ln x$$

D and E both check

ans C

$$(21) \quad 3r = -r$$

$$-\frac{r}{3} = r$$

$$-\frac{r^3}{27} + \frac{r^3}{9} + \frac{54r}{3} + 216 = 0$$

$$r^3 + 243r + 2916 = 0$$

$$-9 \left| \begin{array}{ccc|c} 1 & 0 & 243 & 2916 \\ & -9 & 81 & -2916 \\ \hline 1 & -9 & 324 & 0 \end{array} \right.$$

$$r = -9$$

ans C

$$\textcircled{22} \quad \tan 3x = \tan(2x+x)$$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

Ans C

$$= \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan^2 x}{1 - \tan^2 x}}$$

$$= \frac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x}$$

$$= \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

$$\textcircled{23} \quad f(x+1) = \frac{2f(x)+1}{2}; \quad f(1) = 2$$

$$f(2) = \frac{2+1}{2} = \frac{3}{2}$$

ans. C

$$f(3) = \frac{2 \cdot \frac{3}{2} + 1}{2} = 4$$

$$f(4) = \frac{2 \cdot 4 + 1}{2} = \frac{9}{2}$$

$$f(37) = 2 + 36 \cdot \frac{1}{2} = 20$$



$$(24) f(x) = \sqrt{3} \cos x + 3 \sin x$$

$$A = \sqrt{12} = 2\sqrt{3}$$

$$B = \frac{2\pi}{\frac{2\pi}{3}} = 3$$

$$AB = 6\sqrt{3}$$

ans. B

$$(25) y = \log_e \frac{1 + \sqrt{1-x^2}}{x}$$

$$e^y = \frac{1 + \sqrt{1-x^2}}{x}$$

$$f^{-1}: e^x = \frac{1 + \sqrt{1-y^2}}{y}$$

$$e^x y = 1 + \sqrt{1-y^2}$$

$$e^x y - 1 = \sqrt{1-y^2}$$

$$e^{2x} y^2 - 2e^x y + 1 = 1 - y^2$$

$$(e^{2x} + 1)y^2 - 2e^x y = 0$$

$$y [(e^{2x} + 1)y - 2e^x] = 0$$

$$y \neq 0$$

$$y = \frac{2e^x}{e^{2x} + 1}$$

$$y = \frac{2}{e^x + e^{-x}}$$

ans. B