FUNCTIONS
1989 MAΘ NATIONAL CONVENTION

1) If $h(x) = |x| + |x-1|$, find the range of h.

a) $(-\infty,1)$ b) $(-\infty,1]$ c) $(1,\infty)$ d) $[1,\infty)$ e) nota

2) Given $g(x) = |x-2| - |x| + 2$, express g(x) without absolute value bars if x is in the interval $[0,2)$.

a) $g(x)=4-2x$ b) $g(x)=4$ c) $g(x)=0$ d) $g(x)=2x$ e) nota

3) Find the equation of the oblique asymptote (if it exists) of $g(x)=\frac{x^2-25}{x-4}$.

a) $y=x+5$ b) $y=x-4$ c) $y=x+4$ d) does not exist e) nota

4) $f(x)=\frac{x+1}{x^2+x-6}$, then f(x) is symmetric with respect to:

a) y-axis b) origin c) x-axis d) line $y=x$ e) nota

5) The function f defined by $f(x) = \frac{x+5}{x+k}$ where k is a constant, is one-to-one. Find the value of k so that f will be its own inverse function.

a) 5 b) -5 c) 1 d) -1 e) nota

6) One day on a college campus when there were 5000 people in attendance, a particular student heard that a certain speaker was going to make an unscheduled appearance. This information was told to friends, who in turn related it to others. After t minutes elapsed, f(t) people had heard the rumor, where $f(t)=\frac{5000}{1+4999e^{-0.5t}}$. How many people will eventually hear the rumor?

a) 1000 b) 4000 c) 5000 d) ∞ e) nota

7) Of the six trigonometric functions, how many are even?

a) 1 b) 2 c) 3 d) 4 e) nota

8) Find x, if $e^{2x+2} + e^{x+2} = 6e^2$.

a) $\ln 2$ b) $\ln 3$ c) $\ln 2$ and $\ln 3$ d) $-\ln 2$ e) nota

9) If $f(x)=10^x + 10^{-x}$ and $g(x) = 10^x - 10^{-x}$, find, in simplest form, $[f(x)]^2 + [g(x)]^2$.

a) $20^{2x}+20^{-2x}$ b) $(10^{2x}-10^{-2x})^2$ c) $2(10^{2x}-10^{-2x})$

d) $2(10^{2x}+10^{-2x})$ e) nota

10) Which of the following statements are true for every function f?

I. If $f(x_1)=f(x_2)$, then $x_1=x_2$.

II. If $x_1=x_2$, then $f(x_1)=f(x_2)$.

III. A horizontal line test meets the graph of f in no more than 1 point.

a) I only b) I and II only c) III only d) I and III only e) nota

11) A function is defined by $f(x)=x^2-2x+4$. Find the most extensive subset of the positive real numbers that can be the domain of f, if f is to be a one-to-one correspondence.

a) $x \geq 4$ b) $x \leq 4$ c) $x \geq 1$ d) $x \leq 1$ e) nota

12) Solve: $f(x) > 0$, if $f(x)=x^4-3x^3+x^2+4$.

a) all real numbers b) $x \neq 2$ c) $x > 2$ d) $x < 2$ e) nota

13) f is a function such that $f(x+y)=f(x)\cdot f(y)$, and an x exists for which $f(x)\neq 0$. Find f(0).

a) 0 b) 1 c) -1 d) cannot be determined e) nota

14) If $f(x)=\log_2(4-x^2)$ and $g(x)=\log_2\sqrt{4-x^2}$, what is the relationship between the ordinates of the points in the graphs of f and g for any x in the domain of f and g?

a) ordinates of f are ½ those of g [redacted]
c) ordinates of g are twice those of f
d) ordinates of f are twice those of g e) nota

15) Find the center of symmetry of the function having equation $f(x)=x^3+6x^2+14x+15$.

a) (2,-3) b) (-2,3) c) (3,-2) d) (-3,2) e) nota

16) Which of the following are true?

I. A fourth-degree polynomial function may have exactly three imaginary roots.

II. The product of all the imaginary roots of a polynomial function may be a negative number.

III. A fourth-degree polynomial function may have exactly one real root.

a) I only b) II only c) III only d) II and III only e) nota

17) If $f(x)=2x^2+3x-1$, find $f(\bar{z})$, where z is any complex number.

a) $(2a^2-2b^2+3a-1)-(4ab+3b)i$ b) $(2a^2-2b^2+3a-1)+(4ab+3b)i$

c) $(2a^2+2b^2+3a-1)-(4ab+3b)i$ d) $(2a^2+2b^2+3a-1)+(4ab+3b)i$

e) nota

18) A quadratic function F is such that $F(x) > 0$ if and only if $0 < x < 8$, and F has a maximum value of 32. Find F(x).

a) x^2+8x b) $-x^2-8x$ c) $-2x^2+16x$ d) $2x^2-16x$ e) nota

19) Find the value of k so that the roots of the function $(x-2)(4-x)=x+k$ shall be equal.

a) $\frac{3}{2}$ b) $-\frac{3}{2}$ c) $\frac{7}{4}$ d) $-\frac{7}{4}$ e) nota

20) Find the value of k for which the function $f(x)=x^2-5x+k$ has a chord 3 units long on the x-axis.

a) 1 b) 3 c) 4 d) 5 e) nota

21) If $f(x)=x^2-5x+k$ and the difference of the roots of $f(x)=0$ is 1, find the roots.

a) 3,2 b) -3,-2 c) 5,6 d) -5,-6 e) nota

22) A picture 5 ft. high is placed on a wall with its base 4 ft. above the level of the eye of an observer. Let θ be the radian measure of the angle subtended at the observer's eye by the picture when the observer is x ft. from the wall. Define θ as a function of x.

a) $\theta(x)=\tan^{-1}\frac{9}{x}$ b) $\theta(x)=\tan^{-1}\frac{4}{x}$ c) $\theta(x)=\tan^{-1}\frac{9}{x}+\tan^{-1}\frac{4}{x}$

d) $\theta(x)=\tan^{-1}\frac{9}{x}-\tan^{-1}\frac{4}{x}$ e) nota

23) The cable of a suspension bridge hangs in the form of a parabola when the load is uniformly distributed horizontally. The distance between the two towers is 150m, the points of support of the cable on the towers are 22m above the road, and the lowest point on the cable is 7m above the road. Find the vertical distance to the cable from a point in the road 15m from the foot of a tower.

a) $\frac{48}{5}$ b) $\frac{83}{5}$ c) $\frac{38}{5}$ d) 22 e) nota

24) In $\triangle$ ABC, a=6", b=8". State the range of the area function $\{(x, K(x))\}$ if the area K of the triangle has angle C containing x°.

a) $\{x : 0 < x < 180^\circ\}$ b) $\{x : 0 < x \leq 90^\circ\}$ c) $\{K(x) : 0 < K(x) \leq 24\}$
d) $\{K(x) : 0 < K(x) < 24\}$ e) nota

25) A function F is defined by $F(x) = 2^{|x|}$. Find the axis of symmetry (if there is one) of the graph of F.

a) x-axis b) y-axis c) origin d) no axis of symmetry
e) nota

26) Gas leaks from a container in such a way that at the end of each minute there is $\frac{2}{3}$ as much gas in the container as at the beginning of that minute. What fraction of the original gas remains in the container after x minutes?

a) $1-(\frac{1}{3})^x$ b) $(\frac{1}{3})^x$ c) $1-(\frac{2}{3})^x$ d) $(\frac{2}{3})^x$ e) nota

27) If f is defined by $f(x) = \sin(20^\circ - x) + \sin(40^\circ + x)$, what is the minimum value of f?

a) -1 b) 0 c) 1 d) cannot be determined e) nota

28) Given $f(x) = \begin{cases} x & \text{if } x < 1 \\ x^2 & \text{if } 1 \leq x \leq 9 \\ 27\sqrt{x} & \text{if } 9 < x \end{cases}$, find $f^{-1}(x)$.

a) $f^{-1} = \begin{cases} x, & x < 1 \\ x^{\frac{1}{2}}, & 1 \leq x \leq 81 \\ \left(\frac{x}{27}\right)^2, & x > 81 \end{cases}$

b) $f^{-1} = \begin{cases} x, & x < 1 \\ x^2, & 1 \leq x \leq 9 \\ 27\sqrt{x}, & 9 < x \end{cases}$

c) $f^{-1} = \begin{cases} x, & x < 1 \\ x^2, & 1 \leq x \leq 9 \\ \left(\frac{x}{27}\right)^2, & x > 81 \end{cases}$

d) $f^{-1} = \begin{cases} x, & x < 1 \\ x^{\frac{1}{2}}, & 1 \leq x \leq 9 \\ 27\sqrt{x}, & x > 81 \end{cases}$

e) nota

29) A decorator designs and sells wall fixtures and can sell at a price of $75 each for all the fixtures she produces. If x fixtures are manufactured each day, then the number of dollars in the daily total cost of production is $P(x)=x^2+25x+96$. How many fixtures should be produced each day in order for the decorator to have the greatest profit?

a) 20 b) 25 c) 50 d) 100 e) nota

30) If $f(x)=(x+2)^3$ and $g(x)=\frac{x+5}{x-1}$, find $f^{-1}(g^{-1}(x))$.

a) $f^{-1}(g^{-1}(x))=\left(\frac{3x+4}{x-1}\right)^3$ b) $f^{-1}(g^{-1}(x))=\frac{(x+2)^3+5}{(x+2)^3-1}$

c) $f^{-1}(g^{-1}(x))=\sqrt[3]{\frac{x+5}{x-1}}-2$ d) $f^{-1}(g^{-1}(x))=\sqrt[3]{\frac{x-1}{x+5}}-2$

e) nota