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1999 National Mu Alpha Theta Convention

Limits and Derivatives
Topic Test

1. Evaluate: $\lim_{x\to -2} \frac{|3x-5|}{5x-3}$

A. $-\frac{11}{13}$ B. $-\frac{1}{13}$ C. -1 D. $\frac{11}{13}$ E. NOTA

2. Evaluate: $\lim_{n\to\infty}\left[2-\frac{1}{n}\right]^5$

A. 1 B. 2 C. e^5 D. 32 E. NOTA

3. If $y = \frac{1}{3x-2}$, then find $\frac{d^2y}{dx^2}$.

A. $\frac{1}{(3x-2)^2}$ *B.* $\frac{-3}{(3x-2)^2}$ *C.* $\frac{6}{(3x-2)^3}$ *D.* $\frac{18}{(3x-2)^3}$ *E. NOTA*

4. Evaluate: $\lim_{x\to\infty}\left(\frac{2x^2-x}{x^2-4}\right)^{\frac{1}{x}}$

A. 0 B. 0.5 C. 1 D. 2 E. NOTA

5. Which of the following is the limit of the sequence of partial sums for the series:
$500 + 100 + 20 + 4 + 0.8 + \ldots\ldots$?

A. 0 B. 0.16 C. 624.96 D. 625 E. NOTA

6. If $f(x) = x^{\frac{3}{4}}$, then the derivative of $f^{-1}(x)$ at $x = 27$

A. -81 B. 3 C. 4 D. 81 E. NOTA

7. Evaluate: $\lim_{x\to\frac{3}{2}} \frac{8x^3-27}{2x^2+7x-15}$

A. 0 B. $\frac{36}{13}$ C. $\frac{54}{13}$ D. $\frac{27}{6}$ E. NOTA

8. Which of the following limits does NOT equal 0?

A. $\lim_{x\to 0}\frac{x}{\cos x}$ *B.* $\lim_{n\to\infty}\frac{x^n}{n!}$ *C.* $\lim_{x\to\infty}\frac{\ln x}{x}$ *D.* $\lim_{n\to\infty} 12\left(\frac{3}{5}\right)^n$ *E. NOTA*

9. Evaluate: $\lim_{h\to 0}\dfrac{5(x+h)^3-2(x+h)-5x^3+2x}{h}$

A. $10x^2 - 2x$ B. $15x^2 - 2$ C. $15x^2 + 15xh + 5h^2 - 2$ D. No Limit E. NOTA

10. How many terms of the sequence: $a_n = \dfrac{2n+11}{5n}$ where n ∈ {The Natural Numbers} are **inside** the neighborhood $\langle 0.41,\ 0.42\rangle$?

A. 109 B. 110 C. 219 D. 220 E. NOTA

11. For $f(x) = x^3 + 3x^2 - 9x - 22$, (a, b) is the relative maximum point, (c, d) is the relative minimum point and (e, f) is the inflection point. Find the value of: $3a + 3b + 2c + 2d + e + f$.

A. -8 B. -58 C. -86 D. -238 E. NOTA

12. If $f(x) = \sin x$, $g(x) = \ln x$ for $x > 0$ and $h(x) = f(x)\cdot g(x)$, then which of the following represents $h'(x)$ for all $x>0$?

A. $\dfrac{x\cdot\cos x\cdot\ln x - \sin x}{x}$ B. $(\cos x\cdot\ln x)+1$ C. $\dfrac{x\cdot\cos x\cdot\ln x + \sin x}{x}$

D. $\dfrac{\sin x - x\cdot\cos x\cdot\ln x}{x}$ E. NOTA

13. Evaluate: $\lim_{x\to 0}\dfrac{\tan 7x}{\sin 2x}$

A. No Limit B. $\dfrac{7}{2}$ C. $\dfrac{49}{4}$ D. $\dfrac{2}{7}$ E. NOTA

14. If $f(x) = \dfrac{\sqrt{x+1}}{x-1}$, which of the following represents $f'(x)$?

A. $\dfrac{-x-3}{2\sqrt{x+1}}$ B. $\dfrac{-x-3}{2(x-1)^2\sqrt{x+1}}$ C. $\dfrac{3x}{(x-1)^2}$ D. $\dfrac{3x}{2(x-1)^2\sqrt{x+1}}$ E. NOTA

15. Evaluate: $\lim_{n\to 0^+}\dfrac{\sqrt{n}}{\sqrt{9+\sqrt{n}}-3}$

A. 0 B. 1 C. 3 D. 6 E. NOTA

16. The slope of the line which is tangent to $f(x) = \dfrac{2x+1}{x^2-9}$ at $x = -2$ can be written as a fraction in lowest terms as $\dfrac{A}{B}$. The value of $A + B$ is which of the following?

A. 3 B. 23 C. 27 D. 47 E. NOTA

17. Given $f(x) = \begin{cases} \dfrac{x^2-16}{x-4} & \text{if } x > 4 \\ x+3 & \text{if } x = 4 \\ \left[48 \cdot \left(\dfrac{x-4}{x^3-64}\right) + 7\right] & \text{if } x < 4 \end{cases}$

Which of the following statements are true about $f(x)$?

I. $f(x)$ is defined at $x = 4$

II. the $\lim_{x \to 4} f(x)$ exists

III. $f(x)$ is continuous at $x = 4$

IV. $f'(x)$ is defined at $x = 4$

A. I, II, III only B. I only C. II only D. I and II only E. NOTA

18. If $f(x) = \dfrac{g(x)}{h(x)} - \dfrac{h(x)}{g(x)}$, $g(0) = 5$, $g'(0) = 3$, $h(0) = 1$ and $h'(0) = 4$, find $f'(0)$.

A. $\dfrac{-34}{25}$ B. $\dfrac{-102}{5}$ C. $\dfrac{-442}{25}$ D. $\dfrac{-17}{25}$ E. NOTA

19. Evaluate: $\lim_{h \to 0} \dfrac{2^{3+h} - 8}{h}$

A. 0 B. 2 ln 3 C. 8 D. 8 ln 2 E. NOTA

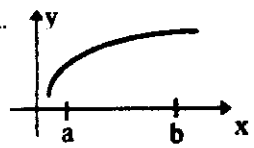
20. Given: $F(x) = f(g(x))$, $g(5) = 12$, $g'(5) = 7$, $f'(5) = 4$ and $f'(12) = 9$. Find $F'(5)$.

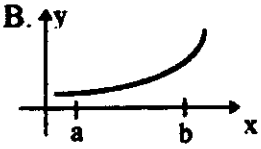
A. 108 B. 84 C. 63 D. 28 E. NOTA

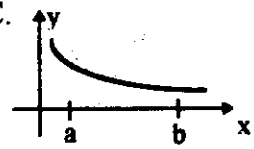
21. The y-intercept of the line normal to the curve $x^2 + xy - y^2 = 1$ at the point (2, 3)?

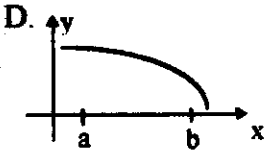
A. $\dfrac{29}{7}$ *B.* $\dfrac{-4}{7}$ *C.* $\dfrac{7}{4}$ *D.* $\dfrac{-1}{2}$ *E. NOTA*

22. For the function f(x) over the interval $a \le x \le b$, $f'(x) > 0$ and $f''(x) < 0$. Which of the following curves could f(x) be represented by ?

A. 

B. 

C. 

D. 

E. NOTA

23. Given that x and y are differentiable functions of t and that $\dfrac{d^2y}{dx^2} = \tan t$, $\dfrac{dy}{dx} = \cos t$. Find $\dfrac{dx}{dt}$.

A. $\sin t$ B. $\cot t$ C. $\sec t$ D. $-\cos t$ E. NOTA

24. If $\lim_{x \to 0} (1+2x)^{\frac{1}{4x}} = e^{\frac{A}{B}}$, then the value of A - B is which of the following?

A. -1 B. 0 C. 1 D. 2 E. NOTA

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25. When simplified, $\lim_{(x,y)\to(\frac{3\pi}{2},B)} \cos(x+y)$ can be written as which of the following?

A. cosB B. sinB C. -sinB D. cosB - sinB E. NOTA

26. A particle moves along the parabola $y = 2x^2$ in the first quadrant in such a way that its x-coordinate increases at a steady 6 m/sec. How fast is the angle of inclination θ of the line joining the particle to the origin changing when x = 2 m ?

A. 12 rad/sec B. 3 rad/sec C. $\frac{3\sqrt{17}}{17}$ rad / sec D. $\frac{12}{17}$ rad / sec E. NOTA

27. Given: $y = \sqrt{x+\sqrt{1+\sqrt{x}}}$, then $\frac{dy}{dx}$ at x = 9 is ?

A. $\frac{\sqrt{11}}{22}$ *B.* $\frac{5\sqrt{11}}{88}$ *C.* $\frac{25\sqrt{11}}{528}$ *D.* $\frac{\sqrt{11}}{44}$ *E. NOTA*

28. Evaluate: $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

A. 0 B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. 1 E. NOTA

29. The function $f(x) = x^4 - 8x^3 + x^2 + 66x - 46$ has exactly one root in the interval $3 < x < 4$. The number 4 is used as the first approximation for this root in Newton's Method. After this first approximation, the root can be written as 3.abcd... where a, b, c and d represent integers 0 through 9. Find a + b + c.

A. 16 B. 17 C. 18 D. 19 E. NOTA

30. A farmer has 600 meters of fencing. He wishes to construct N number of equal size rectangular pens by

running two fences from east to west and the necessary number of fences from north to south. The fences are to be constructed in such a way so that the total area of the pens is a maximum. Once constructed, if one pen is singled out, what is the length in meters of the east to west fence section on the north side of this pen?

A. $\frac{600}{N}$ m B. $\frac{300}{N}$ m C. $\frac{150}{N}$ m D. $\frac{75}{N}$ m E. NOTA

Tiebreakers

T1. Given f(x) = Arcsinx, $g(x) = \sqrt{x}$, $h(x) = 1 - x^2$, k(x) = 3x and ϕ = f(g(h(k(x))))). Find ϕ' ($\frac{1}{6}$)

T2. Find the values of x for which $y = x^4 - 2x^3 - 12x^2$ is concave upward.

T3. Find the limit: $\lim_{x\to 2}\frac{4-x^2}{3-\sqrt{x^2+5}}$