Mu Alpha Theta National Convention: Seattle, 1997
Calculus Limits and Derivatives Topic Test

1. If $f(x) = 2\left(7^x\right)$, determine $f'(2)$

(A) 98 (B) $98 \ln 7$ (C) $49(\ln 7 - 1)$ (D) 49 (E) NOTA

2. Evaluate: $\lim_{x \to \infty} \sqrt[5]{8 + x^4}$

(A) ∞ (B) 0 (C) 1 (D) $\frac{4}{5}$ (E) NOTA

3. Determine $\frac{dy}{dx}$ if $y = 3t^2$ and $x = 5t$

(A) $y = \frac{6}{5}x^{\frac{3}{2}}$ (B) $y = \frac{3}{5}x^{\frac{3}{2}}$ (C) $y = \frac{3}{25}x$ (D) $y = \frac{6}{25}x$ (E) NOTA

4. If $f(x) = \log_x 2x$, what is the value of $f'(8)$?

(A) $\frac{1}{72 \ln 2}$ (B) $\frac{\ln 2}{8}$ (C) $\frac{8}{\ln 2}$ (D) $\frac{\ln 2}{72}$ (E) NOTA

5. Evaluate: $\lim_{n \to 0} (n+1)^{\frac{1}{n}}$

(A) $\frac{1}{e}$ (B) e (C) 2e (D) e^2 (E) NOTA

6. Evaluate: $\lim_{x \to 3} \frac{[x] - 4x}{6}$

(A) $-\frac{5}{3}$ (B) $-\frac{3}{2}$ (C) $-\frac{1}{6}$ (D) $-\frac{1}{2}$ (E) NOTA

7. If $g(x) = \frac{2\left(f(x)\right)^2 + 9}{f(x)}$, $f(7) = 3$, and $f'(7) = -4$, determine $g'(7)$.

(A) 4 (B) 2 (C) -2 (D) -4 (E) NOTA

8. If $f(x) = 4\tan^2(3x^2)$, determine $f'\left(3^{-\frac{1}{2}}\right)$.

(A) $24\sqrt{3}\tan 1\sec^2 1$ (B) $8\sqrt{3}\tan 3\sec^2 3$
(C) $24\sqrt{3}\tan 3\sec^2 1$ (D) $16\sqrt{3}\tan 1\sec^2 1$ (E) NOTA

9. Determine the sum of a and b given that f(x) is continuous and differentiable for all x.

$$f(x) = \begin{cases} 4x - 4, & x \le 8 \\ ax^2 + bx + 8, & x > 8 \end{cases}$$

(A) $\frac{5}{4}$ (B) $\frac{19}{16}$ (C) $\frac{9}{8}$ (D) $\frac{17}{16}$ (E) NOTA

10. What is the slope of the tangent to the locus $(x-2)^2 + \frac{(y+7)^2}{9} = 4$ when $x = 4$ and $y = 3\sqrt{3} - 7$?

(A) $-\sqrt{3}$ (B) $\sqrt{3}$ (C) $-\frac{\sqrt{3}}{3}$ (D) $\frac{\sqrt{3}}{3}$ (E) NOTA

11. Evaluate: $\lim_{x\to 0} \cos\left(\frac{5}{2x}\right)$

(A) 0 (B) 1 (C) $\frac{\sqrt{2}}{2}$ (D) ∞ (E) NOTA

12. At what value of x does the maximum of $f(x) = x^6 e^{-2x}$ occur?

(A) 3 (B) 6 (C) 2 (D) 1 (E) NOTA

13. Evaluate: $\lim_{x\to 0} \frac{3\sin^3 x}{x^3 - 3x^2}$

(A) 1 (B) 3 (C) 0 (D) ∞ (E) NOTA

14. What is the coefficient of the x^3 term in the Taylor expansion of $\tan x$ about $x = 0$?

(A) 0 (B) $\frac{8}{3}$ (C) 2 (D) -2 (E) NOTA

15. If $y = 4x^3 - 2\ln x$ for $x > 0$, determine $y'''(2)$.

(A) 12 (B) $\frac{47}{4}$ (C) $\frac{47}{2}$ (D) 24 (E) NOTA

16. If $f(x) = x^5(2x+3)^4$, what is $f'(-1)$?

(A) -9 (B) 9 (C) -1 (D) 1 (E) NOTA

17. What are the vertical asymptotes of $f(x) = \frac{x+4}{x^2-4}$

(A) $x = \pm 2$ (B) $x = 2$ (C) $x = -2$ (D) $x = -4$ and $x = 2$ (E) NOTA

18. Evaluate: $\lim_{x\to 0} \frac{5^{2x} - 1}{x}$

(A) $1 - 2\ln 5$ (B) $1 + \ln 5$ (C) $\ln 5$ (D) $\ln 25$ (E) NOTA

19. Evaluate: $\lim_{x\to 0} \frac{\sin x}{\sec x - 1}$

(A) ∞ (B) 2 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

20. Evaluate: $\lim_{x\to\infty} \frac{4x - 9}{\sqrt{8x^2 - 12x + 22}}$

(A) 1 (B) $\sqrt{2}$ (C) ∞ (D) 2 (E) NOTA

21. Determine the equation of the line tangent to the graph of $y = 3x3 + 4x + 2$ at the point (2, 38).

(A) $40x + y = 118$ (B) $40x - y = 42$
(C) $-40x + y = -42$ (D) $-40x - y = -118$ (E) NOTA

22. Evaluate: $\lim_{x\to\infty} \frac{1}{n!}$

(A) $\frac{5}{2}$ (B) $\frac{8}{3}$ (C) $\frac{17}{6}$ (D) $\frac{7}{2}$ (E) NOTA

23. Evaluate: $\lim_{x\to\infty} \frac{8x^2 - 16x}{2x^2 - 5x + 12}$

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

24. Evaluate: $\lim_{x\to 3} \frac{2x^2 - 5x - 3}{\sqrt{x} - \sqrt{3}}$

(A) $5\sqrt{3}$ (B) $8\sqrt{3}$ (C) $14\sqrt{3}$ (D) $18\sqrt{3}$ (E) NOTA

25. Evaluate: $\lim_{h\to 0} \frac{\sin(\frac{\pi}{4} - h) - \sin\left(\frac{\pi}{4}\right)}{h}$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $-\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) NOTA

26. Evaluate: $\lim_{x\to 3} \frac{4x^2 - 36}{2x - 6}$

(A) 4 (B) 9 (C) 12 (D) 15 (E) NOTA

27. If $f(x) = 7\pi^4$, what is $f'(x)$?

(A) $28\pi^3$ (B) $7\pi^3 \ln \pi$ (C) $28\pi^3 \ln \pi$ (D) $7\pi^3$ (E) NOTA

28. A box is to be constructed by cutting square corners from a 10 by 6 piece of cardboard and folding the resulting flaps up, taping the corners to create the box. What should the edge length of the square pieces be, to yield a box of maximum volume?

(A) $4 - 2\sqrt{3}$ (B) $1 + \sqrt{2}$ (C) $4 - \sqrt{10}$ (D) $8 - 2\sqrt{13}$ (E) NOTA

29. Determine $y'(e)$, given that $y = x^{\ln x}$ for $x > 0$.

(A) $\frac{4e}{5}$ (B) $e - \frac{1}{e}$ (C) $\ln 7$ (D) 2 (E) NOTA

30. How fast, in cubic centimeters per second, is the volume of a cube increasing if its edges are 6 centimeters long and are lengthening at a rate of 2 cm/s?

(A) 216 (B) 108 (C) 54 (D) 432 (E) NOTA