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1999 Integration Test

Mu Alpha Theta

1) The area of the region enclosed by the graphs of $y = e^x$, $y = 1$, and $x = 1$, is

(A) e

(B) e - 1

(C) e - 2

(D) $\sqrt{e} - 1$

(E) NOTA

2) $\int_0^2 e^{4x}dx =$

(A) $\frac{1}{4}(e^8 - 1)$

(B) $\frac{1}{2}(e^8 - 1)$

(C) $e^8 - 1$

(D) $\frac{1}{4}e^8$

(E) NOTA

3) If $0 < k < \pi$, then $\int_0^k \cos(2x)\,dx = \frac{1}{2}$ when k =

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{12}$ (D) $\frac{3\pi}{4}$ (E) NOTA

4) The region in the first quadrant bounded by $y = \cos x$, $y = \sin x$, the y - axis, and the line $x = \frac{\pi}{4}$ is rotated about the x - axis. The volume of the resulting solid is

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{1}{2}$ (D) $\pi\sqrt{2} - 1$ (E) NOTA

5) $\int_0^3 \frac{x}{x+1}dx$

(A) $2\ln 2$
(B) $6\ln 2$
(C) $3-2\ln 2$
(D) $3+2\ln 2$
(E) NOTA

6) Let $f(x)$ be the function defined by $f(x)=\begin{cases} x, & x \le 0 \\ x+1, & x>0 \end{cases}$,

The value of $\int_{-2}^{1} x\, f(x)dx =$

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) $\frac{7}{2}$ (D) $\frac{11}{2}$ (E) NOTA

7) $\int_e^{e^2} \frac{dx}{x(\ln x)^3} =$

(A) $\int_1^2 \frac{dy}{y^3}$ (B) $-\int_1^2 \frac{dy}{y^3}$ (C) $\int_e^{e^2} \frac{dy}{y^3}$ (D) $\int_1^2 y^3 dy$ (E) NOTA

8) $\int_{-3}^{3} \frac{dx}{x^2+9} =$

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) NOTA

9) $\int_2^4 |x-3|dx =$

(A) 1 (B) -1 (C) 0 (D) -2 (E) NOTA

10) The average value of the function $f(x)=\sin x$ on the closed interval $\left[0,\frac{\pi}{2}\right]$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) 1 (D) $\frac{2}{\pi}$ (E) NOTA

11) $\int_{e}^{e^2} \ln x \, dx =$

(A) e (B) e^2 (C) $e^{-2} - e^{-1}$ (D) $\frac{3}{2}$ (E) NOTA

12) If $\int_0^2 (2x^3 - kx^2 + 2k)dx = 12$, then k must be

(A) 1 (B) 2 (C) 3 (D) -2 (E) NOTA

13) Which definite integral represents the volume of a sphere with radius 2?

(A) $\pi\int_{-2}^{2}(x^2 - 4)dx$

(B) $\pi\int_{-2}^{2}(x^2 + 4)dx$

(C) $2\pi\int_{0}^{2}(4 - x^2)dx$

(D) $2\pi\int_{-2}^{2}(4 - x^2)dx$

(E) NOTA

14) Suppose $G(x) = \int_0^x \frac{1}{1+t^3}dt$ for all real x, then $G'(1) =$

(A) 1 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) NOTA

15) $\int xe^{3x}dx =$

(A) $\frac{1}{3}e^{3x}\left(x - \frac{1}{3}\right) + C$

(B) $\frac{1}{3}e^{3x}(x - 1) + C$

(C) $e^{3x} - \frac{1}{3}e^{3x} + C$

(D) $\frac{1}{3}e^{3x}\left(x + \frac{1}{3}\right) + C$

(E) NOTA

16) For what value of k, $k > 0$, does $\int_0^k (4kx - 5k)\,dx = k^2$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

17) Suppose that the area under the curve $y = \frac{1}{x}$ from $x = a$ to $x = b$ is k.

The area, in terms of k, under the curve $y = \frac{1}{x}$ from $x = 2a$ to $x = 2b$ is

(A) k (B) $2k$ (C) k^2 (D) $k + 2$ (E) NOTA

18) The region enclosed by the line $x + y = 1$ and the coordinate axes is rotated about the line $y = -1$. What is the volume of the solid genenerated?

(A) $\frac{17\pi}{2}$ (B) $\frac{17\pi}{4}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$ (E) NOTA

19) Solve for x: $\int_{x^2}^{x^5} \frac{2}{t}\,dt = 54$

(A) e^9 (B) 9 (C) e^3 (D) 3 (E) NOTA

20) $\int_1^3 \frac{\sin\left(\frac{\pi}{x}\right)}{x^2}\,dx =$

(A) $\frac{\sqrt{3}+2}{2\pi}$ (B) $\frac{\pi}{2}$ (C) $\frac{1}{2\pi}$ (D) $\frac{3}{2\pi}$ (E) NOTA

21) If n is a positive integer, then $\int_0^{n\pi} |\sin x|\,dx$

(A) $2n$ (B) $-2n$ (C) $2n(\pi - 1)$ (D) n (E) NOTA

22) $\int_2^4 \frac{x^2+2}{4x} dx =$

(A) $\frac{3+\ln 2}{2}$ (B) $\frac{1+\ln 2}{2}$ (C) $\frac{9}{2}$ (D) $\frac{5}{2}$ (E) NOTA

23) The region bounded by the x - axis and the part of the graph of $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$ is divided into two regions by line $x = c$. If the area for the region for $0 \le x \le c$ is equal to the area of the region for $c \le x \le \frac{\pi}{2}$, then c must be

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{9}$ (E) NOTA

24) If the region bounded by the curve $f(x) = \sec x$, the x - axis, y - axis, and the line $x = \frac{\pi}{4}$, is revolved about the x - axis, what is the volume of the resulting solid?

(A) $\frac{\pi}{6}$ (B) π (C) 2π (D) $\frac{\pi}{2}$ (E) NOTA

25) If $f(x)$ is continuous on the interval $[a,b]$ and if $\int_a^x f(t)\,dt = 0$ for all x in $[a,b]$, then which of the following must be true?

I. f is constant on $[a,b]$
II. $f(x) \ge 0$ for all x in $[a,b]$
III. $f(x) = 0$ for all x in $[a,b]$

(A) I only (B) II only (C) I and II only
(D) I, II, and III (E) NOTA

26) Which of the following is equal to $\int \frac{dx}{16+x^2}$?

(A) $\text{Arc tan } 4x + c$ (B) $\text{Arc tan } \frac{x}{4} + c$ (C) $\frac{1}{4}\text{Arc tan } \frac{x}{4} + c$

(D) $\frac{1}{4}\text{Arc tan } 4x + c$ (E) NOTA

27) If $\int x \sec^2 x \, dx = f(x) + \ln |\cos x| + c$, then $f(x) =$

(A) $\tan x$ (B) $\frac{1}{2}x^2$ (C) $x \tan x$ (D) $x^2 \tan x$ (E) NOTA

28) The Area of the region between the graph of
$y = 3x^2 + 3$ and the x - axis from $x = 1$ to $x = 3$ is

(A) 36 (B) 32 (C) 27 (D) 24 (E) NOTA

29) $\int_0^{\frac{\pi}{2}} \cos^2 x \sin x \, dx =$

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) NOTA

30) $\int \frac{e^{\frac{-1}{x^2}}}{x^3} dx =$

(A) $\frac{e^{\frac{-1}{x^2}}}{2} + C$ (B) $e^{\frac{-1}{x^2}} + C$ (C) $\frac{-2e^{\frac{1}{x}}}{x^2} + C$

(D) $\frac{e^{\frac{2}{x^3}}}{x^4} + C$ (E) NOTA

Integration

Tie Breakers

T.1 $\int \frac{dx}{(x-1)(x+2)} =$

T.2 If $f(x) = \frac{x}{\tan x}$, then $f'\left(\frac{\pi}{4}\right) =$

T.3 The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is