MU ALPHA THETA 1994

Topic Test

Calculus

1. Let $f(x) = 4x^3 - x^4$. Then $f''(x) =$

   A. $4x - x^2$ B. $12x^2(1 - 4x)$ C. $12x(2 - x)$
   D. $x(4 - x)$ E. $x^3(4 - x)$

2. Find the relative maximum of $f(x) = 4x^3 - x^4$ on the interval $[0, 4]$.

   A. 27 B. 36 C. 0
   D. 1 E. 3

3. Let $f(x) = x^2 - 2x$ and $m(x)$ denote the slope of the line segment from $(1, -1)$ to $(x, f(x))$ for $x \neq 1$. Then for $x \neq 1$, $m(x) =$

   A. $x^2 - 2x + 1$ B. $x - 1$ C. $2(x - 1)$
   D. $x^2 - 2$ E. $2x - 1$

4. Let $f(x) = 1/(x^2 + 1)$. Compute the average rate of change of f over the interval $[2, 4]$.

   A. $-6/85$ B. $-8/17^2$ C. $-4/25$
   D. 0 E. -1

5. Find x so that $\int_1^x 1/(1+t^2)dt = \pi/12$

   A. 2 B. 2π C. $\sqrt{3}/2$
   D. $\pi/3$ E. $\sqrt{3}$

6. Let $f(x) = 4 - x^2$. Find the point(s) $(a, f(a))$ on the graph of f where the tangent line passes through the point $(5/2, 0)$.

   A. Only at (1, 3) B. only at (0, 0) C. only at (4,-12)
   D. At both (1,3) and (4, -12)
   E. at none of the mentioned points

7. If $g(x) = \int_3^x \cos t^2 \, dt$, then $g'(x) =$

   A. $-2x \sin x^2$ B. $\sin x^2$
   C. $\cos x^2 - ((\sin x^2)/(\cos x^2))$
   D. $2x \cos x^2$ E. $\cos x^2$

8. $\lim_{x\to\infty} ([x] - x)/x =$

A. 1 B. -1 C. 0
D. ∞ E. undefined

9. As $n \to \infty$,
$\lim (\sin 1\pi/n + \sin 2\pi/n + \sin 3\pi/n + \cdots + \sin (n-1)\pi/n)\bullet(\pi/n) =$

A. 2 B. 0 C. 1
D. 2π E. π

10. $\lim_{x\to 0+} x \ln x =$

A. 0 B. $-\infty$ C. 1
D. $+\infty$ E. -1

11. Find the volume of the solid of revolution obtained by rotating the planar region enclosed by the graphs of $y = 3 - x^2$ and $y = x + 1$ about the line $x = -2$.

A. $27\pi/2$ B. $9\pi/2$ C. $3\pi/4$
D. 9/2 E. 9π

12. Evaluate $\int_0^1 (4 - x^2)^{-3/2} dx$

A. ∞ B. 3/20 C. $\pi/20$
D. $(15/4)^{3/2}$ E. $\sqrt{3}/12$

13. The function $y = xe^{-2x}$ satisfies which of the following differential equations:

A. $y''-2y'+y = 0$ B. $y''-4y'+4y = 0$ C. $y''+4y'+4y = 0$
D. $y''-4y = 0$ E. $y''+2y'+y = 0$

14. A function $f(x)$ is said to have a *simple zero* at $x = a$; or a *double zero* at $x = a$, accordingly as $f(a) = 0$ and $f'(a) \neq 0$; or $f(a) = f'(a) = 0$ and $f''(a) \neq 0$. Apply these definitions to the function $f(x) = e^x - |x|^e$ and complete the next sentence properly: On the interval $-\infty < x < \infty$, $f(x)$ has

A. only one double zero. B. two different simple zeros.
C. one simple and one different double zero.
D. three simple zeros. E. (None of answers A. through D.)

15. If $f(x) = x^2 - 2$ and a sequence $x_1, x_2, x_3, x_4, \cdots$ is recursively defined by $x_1 = 1$ and $x_{n+1} = x_n - (f(x_n)/f'(x_n))$, compute x_4

A. 17/12 B. 3/2 C. 131/90
D. 1 E. 577/408

16. If $x_1 = 100$ and a sequence is recursively defined by $x_{n+1} = (x_n+6)/3$, then as $n \to \infty$, $\lim x_n =$

A. 3 B. 106/3 C. 124/9
D. 6.59259 E. 3.2

17. The centroid (*i.e., the center of mass of a material with uniformly constant mass per unit area*) of the planar region enclosed by the curves $y = x^2$ and $y = 2x$ is $(x^*, y^*) =$

A. (1, 8/5) B. (1, 3/2) C. (9/8, 2)
D. (7/9, 7/9) E. (1, 2)

18. Two right circular cylinders, each having the same radius of 2 inches, when intersected perpendicularly enclose a solid region with cross sections being squares when sliced in the proper direction. The volume of the enclosed solid is

A. 42 B. 48 C. 13π
D. 128/3 E. $17\pi^2/4$

19. Given that $\lim (1 + 1/n)^n = e$ as $n \to \infty$, use this and appropriate limit laws to evaluate $\lim (1 + 1/(n+1))^{2n}$ as $n \to \infty$

A. $e^{1/2}$ B. e^2 C. $e^{2/3}$
D. $e^{3/2}$ E. e

20. As $n \to \infty$, $\lim (1 + 3/n)^n =$

A. e^3 B. 1 C. $e^{1/3}$
D. e E. e^2