1989 NATIONAL MAΘ INTEGRATION TEST

1) Evaluate: $\int_0^1 e^{x^2}dx + \int_1^e \sqrt{\ln x}\,dx$

a) 1 b) $e/2$ c) e d) 2e e) nota

2) $\int \left(\sqrt{1 + f(x)}\right) f'(x)\,dx = ?$

a) $(2/3)(1+f(x))^{3/2}+c$ b) $(2/3)(1+f'(x))^{3/2} + c$

c) $(1/2)(1+f(x))^{3/2}+c$ d) $(1/3)(1+f(x))^{3/2} + c$

3) If $F(x) = \int_0^{\sin x} (1-t^2)dt$, then $F'(x)$ is :

a) $\sin x - \frac{\sin^3 x}{3}$ b) $1-\sin^2 x$ c) $\cos^3 x$ d) $\sin x\cos^2 x$ e) nota

4) $\int 10^{ax}dx = ?$

a) $a10^{ax} + c$ b) $\frac{-10^{ax}}{a\ln 10} + c$ c) $a\ln 10^{ax} + c$ d) $10\ln 10^{ax}+c$ e) nota

5) $\int_{\pi/2}^{3\pi/2} \sqrt{\cos^2 x}dx=?$

a) -2 b) 0 c) 1 d) 2 e) nota

6) $\int_{-1}^{2} \frac{|x|}{x}\, dx = ?$

a) -3   b) 1   c) 2   d) 3   e) nota

7) If $f(x) = \begin{cases} 8-x^2 & \text{for } -2 \leq x \leq 2 \\ x^2 & \text{for all other } x \end{cases}$, then $\int_{-1}^{3} f(x)\, dx$ is a number between:

a) 0 and 8   b) 8 and 16   c) 16 and 24   d) 24 and 32   e) nota

8) If $f(x) = \int_0^x \frac{dt}{\sqrt{t^3+2}}$, which of the following is FALSE?

a) $f(0)=0$ b) f is continuous for all $x \geq 0$ c) $f(-1) > 0$

d) $f'(1) = \frac{1}{\sqrt{3}}$ e) nota

9) Which of the following integrals gives the length of the graph of y=tanx between x=a and x=b, where $0 < a < b < \pi/2$?

a) $\int_a^b \sqrt{x^2+\tan^2 x}\,dx$ b) $\int_a^b \sqrt{x+\tan x}\, dx$ c) $\int_a^b \sqrt{1+\sec^4 x}\, dx$

d) $\int_a^b \sqrt{1+\tan^2 x}\, dx$ e) nota

10) If n is a known integer, for what value of k is $\int_1^k x^{n-1} dx = \frac{1}{n}$?

a) 0   b) $\left(\frac{2}{n}\right)^{1/n}$ c) 2^n d) $2^{(1/n)}$ e) nota

11) $\int_{\pi/4}^{\pi/2} [\ln(\sin x)](\cot x)\,dx = ?$

a) $\ln(\pi/2)$ b) $\frac{-(\ln 2)^2}{8}$ c) $\left(-\pi/2\right)\ln 2$ d) $-\frac{1}{4}$ e) nota

12) $\int_0^1 (4-x^2)^{-3/2}dx=?$

a) $\frac{2-\sqrt{3}}{3}$ b) $\frac{2\sqrt{3}-3}{4}$ c) $\frac{\sqrt{3}}{12}$ d) $\frac{\sqrt{3}}{3}$ e) nota

13) $\int \text{Arcsin}x\,dx=?$

a) $\sin x - \int \frac{x\,dx}{\sqrt{1-x^2}}$ b) $-\frac{(\text{Arcsin}x)^2}{2} + c$ c) $\text{Arcsin}x + \int \frac{dx}{\sqrt{1-x^2}}$

d) $x\,\text{Arccos}x - \int \frac{x\,dx}{\sqrt{1-x^2}}$ e) nota

14) $\int_0^1 \ln^2 x\,dx = ?$

a) cannot be determined b) 1 c) 2 d) e e) nota

15) If $5x^3+40=\int_c^x f(t)\,dt$, find the value of c.

a) -2 b) 0 c) 1 d) 5 e) nota

16) $\int \frac{x\,dx}{\sqrt{1+x}} = ?$

a) $\frac{2(1+x)^{3/2}}{3} - 2(1+x)^{\frac{1}{2}} + c$ b) $\frac{2(1+x)^{3/2}}{3} + c$

c) $\frac{2(1+x)^{3/2}}{3} + 2(1+x)^{\frac{1}{2}} + c$ d) $\frac{(1+x)^{\frac{1}{2}}}{2} + c$ e) nota

17) If the substitution $\sqrt{x} = \sin(y)$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}\,dx}{\sqrt{1-x}}$, the resulting integral is:

a) $\int_0^{1/2} \sin^2 y\, dy$ b) $2\int_0^{1/2} \frac{\sin^2 y\, dy}{\cos y}$ c) $\int_0^{\pi/4} 2\sin^2 y\, dy$

d) $\int_0^{\pi/4} \sin^2 y\, dy$ e) nota

18) If $\int_1^2 f(x-c)\, dx = 5$, where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx = ?$

a) 5+c   b) 5   c) 5−c   d) c−5   e) nota

19) The length of the curve $y = \ln(\sec x)$ from $x=0$ to $x=b$, where $0 < b < \pi/2$, may be expressed by which of the following integrals?

a) $\int_0^b \sec x\,dx$ b) $\int_0^b \sec^2 x\,dx$ c) $\int_0^b (\sec x \tan x)\,dx$

d) $\int_0^b \sqrt{1+(\ln \sec x)^2}\, dx$ e) nota

20) By the Mean Value Theorem for Integrals (also called the Intermediate Value Theorem), $\int_a^b f(x)\,dx = (b-a)f(c)$.

For the case $f(x)=x$, c is?

a) $\frac{a-b}{2}$ b) $\frac{b-a}{2}$ c) $a+b$ d) $\frac{a+b}{2}$ e) nota

21) $\int_0^1 \frac{x-1}{x+1}\,dx = ?$

a) $1-2\ln 2$ b) $1+\ln 2$ c) $1+2\ln 2$ d) $\ln 2-1$ e) nota

22) Let f be a continuous function defined for all real x and

1) $\int_2^4 f(x)\,dx = 2/3$ 2) $\int_2^7 f(x)\,dx = 11$ Find: $\int_4^7 (3f(x)+5)\,dx$

a) 31/3 b) 49/3 c) 36 d) 46 e) nota

23) Suppose $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \int_0^x t g'(t)\,dt$ for all $x \geq 0$. Which of the following statements is FALSE?

a) F takes on negative values
b) F is continuous for all $x > 0$.
c) $F(x) = x(g(x)) - \int_0^x g(t)\,dt$
d) F is an increasing function
e) nota

24) $\int_{\infty}^{\infty} xe^{-x^2}\,dx = ?$

a) -1 b) 0 c) 1 d) ∞ e) nota

25) $\int \operatorname{sech} x\,dx = ?$

a) $\ln|\operatorname{sech}x + \tanh x| + c$ b) $-\ln|\cosh x| + c$

c) $\arctan(\sinh x) + c$ d) $\dfrac{1}{\sinh x} + c$ e) nota

26) Find the length of the spiral $r=e^{2\theta}$ from $\theta=0$ to $\theta=2\pi$.

a) $5e^{4\pi}$ b) $\sqrt{5}\,e^{4\pi}$ c) $\frac{\sqrt{5}}{2}(e^{4\pi}-1)$ d) $\frac{5}{2}(e^{4\pi}-1)$ e) nota

27) Evaluate $\iint_R dA$ where R is the region in the first quadrant bounded by the semi-cubical parabola $y^2 = x^3$ and the line $y=x$.

a) $\frac{1}{10}$ b) $\frac{1}{2}$ c) $\frac{1}{5}$ d) $\frac{2}{5}$ e) nota

28) $\int_3^6 xy\,dx$, when $x=6\cos\theta$ and $y=2\sin\theta$.

a) 8 b) -8 c) $-9\sqrt{3}$ d) $81\sqrt{3}$ e) nota

29) Let $f(x)$ be a continuous function then $\lim_{n\to\infty}\left[\frac{1}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)\ldots+f\left(\frac{n}{n}\right)\right)\right]$ expressed as an integral is:

a) $\int_0^1 f(x)dx$ b) $\int_0^n f(x)dx$ c) $\int_0^1 f\left(\frac{1}{x}\right) dx$

d) $\int_0^n f\left(\frac{1}{x}\right)dx$ e) nota

30) $f(x)$ and $g(x)$ are odd functions continuous on $[-a,a]$, where $a>0$. $f(-a) = g(-a)$. $F'(x) = f(x)$ and $G'(x) = g(x)$.

Then $\int_{-a}^{a} [f(x)-g(x)]dx = ?$

a) $F(a)-G(a)$ b) $F(-a)-G(-a)$ c) $2[F(a)-G(a)]$ d) 0 e) nota