Mu Alpha Theta National Convention 2004
Mu Individual
For each question, NOTA means none of the above answers

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Answers

#	Answer	#	Answer
1	C	18	D
2	C	19	C
3	B	20	E
4	D	21	B
5	A	22	B
6	B	23	D
7	E	24	E
8	D	25	D
9	A	26	B
10	D	27	Thrown out
11	B	28	A
12	D	29	C
13	E	30	C
14	E	TB1	-145
15	A	TB2	$\pi/2$
16	B	TB3	
17	A		

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1) **(C)** Average value = $\frac{1}{0-(-4)}\int_{-4}^{0}\sin\frac{x}{2}\,dx = \frac{\cos 2 - 1}{2}$.

2) **(C)** To be divisible by 21, the number has to be divisible by 3 and 7; thus $3+7+3+0+N+5=18+N$ has to be a multiple of 3. That means N has to be 0, 3, 6, or 9. It's easy to just check each of these one by one to see if the original number is divisible by 7. It follows that 6 is the only one that works. The sum of the positive integral factors of 6 is 1 + 2 + 3 + 6 = 12.

3) **(B)** Area = $\int_0^2 e^{-x}\,dx = -e^{-x}\Big]_0^2 = -e^{-2}+1$.

4) **(D)** Since $\log_3 27 = 3$ and $10^{\log 3} = 3$, the equation reduces to $\log_5(\log_4(\log_2 x)) = 0$; thus $x = 2^{4^{5^0}} = 2^{4^1} = 16$.

5) **(A)** The degree of the numerator is bigger than the denominator's so as *x* gets larger on the positive scale, the numerator dominates. Furthermore, since the numerator is negative and the denominator positive, the limit is $-\infty$.

6) **(B)** By the Binomial Theorem, $(i+2)^6 = \binom{6}{0}i^6 2^0 + \binom{6}{1}i^5 2 + \binom{6}{2}i^4 2^2 + \binom{6}{3}i^3 2^3 + \binom{6}{4}i^2 2^4 + \binom{6}{5}i^1 2^5 + \binom{6}{6}i^0 2^6$. Using the periodic cycle of powers of *i*, the above simplifies to $-117+44i$.

7) **(E)** Statement 1 is false in general; for example, take $f(x) = |x|$, which is continuous everywhere but is not differentiable at the origin. Statement 2 is true; it's actually a theorem. Statement 3 is also true because $f(x) \ge 3$ implies $\int_4^8 f(x)\,dx \ge \int_4^8 3\,dx = 12 > 9$.

8) **(D)** Complete the square to get $\frac{(x-1)^2}{16} + \frac{(y+1)^2}{8} = 1$; the eccentricity is given by $\frac{\sqrt{16-8}}{\sqrt{16}} = \frac{\sqrt{2}}{2}$.

9) **(A)** We have $A = \pi r^2$, so $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$. Plug in all the known values to obtain $\frac{dA}{dt} = 2\pi(5)(1) = 10\pi$.

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10) **(D)** Combine the fractions and use trig. identities to get
$$\frac{\sin(3x)\cos x-\cos(3x)\sin x}{\sin x\cos x}=\frac{\sin(3x-x)}{(1/2)\sin(2x)}=\frac{2\sin(2x)}{\sin(2x)}=2.$$

11) **(B)**

$$\int_0^{\pi/12}\frac{dx}{\cot(3x)}=\int_0^{\pi/12}\tan(3x)\,dx=\frac{1}{3}\ln|\sec(3x)|\Big]_0^{\pi/12}$$
$$=\frac{1}{3}\ln\left(\sec\frac{\pi}{4}\right)-\frac{1}{3}\ln(\sec 0)$$
$$=\frac{1}{3}\ln 2^{1/2}=\frac{1}{3}\left(\frac{1}{2}\ln 2\right)=\frac{\ln 2}{6}$$

12) **(D)** The odd terms form a geometric series with first term of $\frac{1}{2}$ and common ratio of $\frac{1}{2}$; its sum is $\frac{1/2}{1-1/2}=1$. The even terms form a geometric series whose first term is $\frac{2}{3}$ and common ratio $\frac{1}{2}$; the sum of that is $\frac{2/3}{1-1/2}=\frac{4}{3}$. Altogether the sum is $1+\frac{4}{3}=\frac{7}{3}$.

13) **(E)** Separate variables to obtain $e^{-y}\,dy=xe^{-x^2}\,dx$; thus $-e^{-y}=-\frac{1}{2}e^{-x^2}+C$. Use the fact that the curve passes through the origin to get $C=-\frac{1}{2}$. Thus, when $x=1$, $y=-\ln\left(\frac{1}{2e}+\frac{1}{2}\right)$.

14) **(E)** If *X* is the number of heads, then $P(X=i)=\frac{C(7,i)}{2^7}$, where $C(n,i)=\frac{n!}{(n-i)!i!}$. Thus,
$$P(X\le 5\mid X\ge 2)=\frac{P(2\le X\le 5)}{P(X\ge 2)}=\frac{(C(7,2)+C(7,3)+C(7,4)+C(7,5))/2^7}{(C(7,2)+C(7,3)+C(7,4)+C(7,5)+C(7,6)+C(7,7))/2^7}=\frac{14}{15}.$$

15) **(A)** Take natural logs of both sides to obtain $\ln y=(\ln x)^2$. Differentiate implicitly and we have $\frac{1}{y}\frac{dy}{dx}=\frac{2\ln x}{x}$. Thus, $\frac{dy}{dx}=\frac{2y\ln x}{x}=\frac{2x^{\ln x}\ln x}{x}$.

16) **(B)** Use your favorite technique of solving 3 x 3 systems to get $(x,y,z)=(4,5,1)$; thus $x^2+y^2+z^2=16+25+1=42$.

17) **(A)** We'll get extrema when the derivative is 0 or undefined. The derivative of *f* is $f'(x)=(x-4)(x+3)^2(5x-6)$; it's obviously not undefined, but it equals 0 whenever *x* is –3, 4, or 6/5. Two of these values lie on the interval $-3<x<6$.

18) **(D)** Let *ABCD* be a rectangle with *AB* = 48 and *BC* = 55. Fold the rectangle such that A coincides with *C*. Unfold the rectangle. Let *EF* be the crease, where *E* and *F* are on *AD* and *BC*, respectively. By symmetry, *BF* = *ED* = *x*. Now fold the rectangle again. Notice that the points *A* (which now coincides with *C*), *E*, and *D* make a right triangle with legs *x* and 48 with hypotenuse 55 – *x*. By the Pythagorean theorem, $(55-x)^2 = x^2 + 48^2$; this yields *x* = 721/110. Now unfold the rectangle again. Notice the length of the crease *L* is the hypotenuse of a triangle with legs of 48 and 55 – 2*x*. Thus, $L = \sqrt{48^2 + (55-2x)^2} = \sqrt{48^2 + (55-2(\frac{721}{110}))^2} = \frac{3504}{55}$, making *m* + *n* = 3559.

19) **(C)** By the Shell Method, the volume is $2\pi\int_0^2 (4-x)(8-x^3)\,dx = \frac{384\pi}{5}$.

20) **(E)** (CHANGED TO E) Set A knows; Set B guesses wrong; set C guesses right – 180 answer wrong
→ P(guessing wrong) = 4/5   4/5 (# guessing) = 180   # guessing = 225 →
\# guessing + # who knew = 400 → # who knew = 400 – 225 = 175

21) **(B)** First things first, since the graph passes through the origin, *d* = 0. Now from the information given, $y'(0) = y'(3) = 0$. Since $y' = 3ax^2 + 2bx + c$, this makes $c = 0$ and $0 = 3a(3)^2 + 2b(3) = 27a + 6b$. The graph also passes through (3, 4), so $4 = a(3)^3 + b(3)^2 = 27a + 9b$. Solving this yields $(a,b) = \left(-\frac{8}{27}, \frac{4}{3}\right)$; thus

$$a + b + c + d = -\frac{8}{27} + \frac{4}{3} + 0 + 0 = \frac{28}{27}.$$

22) **(B)** largest number on top / smallest number on bottom   9321/15 = 621.4

23) **(D)** The quickest way to do this problem is probably to use tabular integration:

+/−	u	dv
$+\to$	$(2x+1)^2$]	$\sin(\pi x)$
$-\to$	$8x+4$]	$-\frac{1}{\pi}\cos(\pi x)$
$+\to$	8]	$-\frac{1}{\pi^2}\sin(\pi x)$
$-\to$	0	$\frac{1}{\pi^3}\cos(\pi x)$

Multiply along the arrows to obtain an antiderivative:
$-\frac{(2x+1)^2\cos(\pi x)}{\pi} + \frac{(8x+4)\sin(\pi x)}{\pi^2} + \frac{8\cos(\pi x)}{\pi^3}$. Use the Fundamental Theorem of Calculus to obtain the answer in choice D.

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24) **(E)** $3x + 4x + 5x = 540$ $x = 45$ $135 + 180 + 225 = 540$ $135^2 + 180^2 = 225^2$

25) **(D)** Differentiating implicitly, we get that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$; the curve will have a horizontal tangent line when this quantity is 0. This occurs when $3x^2y - y^2 = y(3x^2 - y) = 0$. If $y = 0$, then we have $x(0)^2 - x^3(0) = 0 = 6$, a contradiction. Thus, $3x^2 = y$; plug this into the original equation to get $9x^5 - 3x^5 = 6$, or $x = 1$, making $y = 3$. The sum of the coordinates is 4.

26) **(B)** Regroup the equation as $\cos x - \cos(3x) - \sin(2x) = 0$. Now use the sum-to-product identities on the first two terms to obtain $-2\sin(2x)\sin(-x) - \sin(2x) = 0$, which simplifies to $\sin(2x)(2\sin x - 1) = 0$. On the interval $[-2\pi, 2\pi]$, $\sin(2x) = 0$ has the solution set $\left\{0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \pm 2\pi\right\}$. The equation $2\sin x - 1 = 0$ has solutions $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}\right\}$. There are 13 solutions in all.

27) **(THROWN OUT)** The formula for the area of a regular dodecagon with side length *s* is $3(2+\sqrt{3})s^2$; one can derive this by breaking up the dodecagon into 12 isosceles triangles with vertex angle of 360/12 = 30 degrees and using some trigonometry. At any rate, since $s = y = \sqrt{x}$, the volume is equal to $\int_0^1 3(2+\sqrt{3})(\sqrt{x})^2\,dx = \frac{3}{2}(2+\sqrt{3})$.

28) **(A)** Recall the factorizations $a^2 + b^2 = (a+b)^2 - 2ab$ and $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$. Since *f* is a quadratic, the sum and product of the roots is given by $-\frac{(-2m)}{1} = 2m = a+b$ and $\frac{m}{1} = m = ab$, respectively. Thus, $r_1^2 + r_2^2 = (2m)^2 - 2(m) = 4m^2 - 2m$ and $r_1^3 + r_2^3 = (2m)^3 - 3(m)(2m) = 8m^3 - 6m^2$. Set these two quantities equal to each other to get $8m^3 - 10m^2 + 2m = 0$. The sum of the solutions to this equation is $-\frac{(-10)}{8} = \frac{5}{4}$.

29) **(C)** Let $A = x^2 + x - 5$ and $B = x^2 + 4x + 3$. Now for $A^B = 1$, we must have either $A = 1$ (and B can be anything), $B = 0$ (in this case $A \neq 0$), or $A = -1$ and B is even. The first case yields x being equal to –3 or 2. The second case yields x being equal to –3 or –1. The third case only produces non-integer values for B. Thus, the sum of all distinct solutions is $s = -3 - 1 + 2 = -2$. We have $f'(x) = (x+2)(x+3) + (x+1)(x+3) + (x+1)(x+2)$, making $f'(s) = 0 + (-1)(1) + 0 = -1$.

30) **(C)**

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Tiebreaker #1 -145

$$\frac{d}{dx}h(x) = h'(f(x))f'(x) = (3(f(x))^2 + 2)f'(x)$$
$$\frac{d}{dx}h(1) = (3(-3)^2 + 2)(-5) = -145$$

Tiebreaker #2 $\frac{\pi}{2}$

$$\int_{-\infty}^{\infty} \frac{\mathrm{dx}}{\mathrm{e}^x + e^{-x}} = \lim_{b\to+\infty}\int_0^b \frac{e^x dx}{e^{2x}+1} + \lim_{a\to-\infty}\int_a^0 \frac{e^x dx}{e^{2x}+1} =$$
$$\lim_{b\to+\infty} \arctan e^x \Big|_0^b + \lim_{a\to-\infty} \arctan e^x \Big|_a^0$$
$$\lim_{b\to+\infty} (\arctan\ e^b - \frac{\pi}{4}) + \lim_{a\to-\infty} (\frac{\pi}{4} - arctan\ e^a) =$$
$$\left(\frac{\pi}{2} - \frac{\pi}{4}\right) + \left(\frac{\pi}{4} - 0\right) = \frac{\pi}{2}$$