Calculus Individual – Mu Level
2000 Mu Alpha Theta National Convention

Note: For each of the following questions, answer E, NOTA, means "None of the Above."

1. $\lim_{h\to 0}\dfrac{\cos\left(\dfrac{\pi}{2}+h\right)}{h}$ is

A. 1 B. 0 C. –1 D. nonexistent E. NOTA

2. $\int_0^1 \dfrac{x\,dx}{x^2+1}$ is equal to

A. $\dfrac{\pi}{4}$ B. $\ln\sqrt{2}$ C. $\dfrac{1}{2}(\ln 2-1)$ D. $\ln 2$ E. NOTA

3. Let $F(x)=\int_0^x \dfrac{10}{1+e^t}dt$. Which of the following statements are true?

I. F '(0) = 5 II. F(2) < F(6) III. F is concave upward for all x.

A. I only B. II only C. III only D. I and II E. NOTA

4. The base of a solid is the region bounded by the parabola $y^2 = 4x$ and the line x = 2. Each plane section perpendicular to the x-axis is a square. The volume of the solid is

A. 6 B. 8 C. 10 D. 16 E. NOTA

5. If $x=\sqrt{1-t^2}$ and $y=\sin^{-1}t$, then $\dfrac{dy}{dx}$ equals

A. $-\dfrac{\sqrt{1-t^2}}{t}$ B. $-t$ C. $\dfrac{t}{1-t^2}$ D. 2 E. NOTA

Calculus Individual – Mu Level
2000 Mu Alpha Theta National Convention

6. The area of the largest isosceles triangle that can be drawn with one vertex at the origin and with the others on a line parallel to and above the x-axis and on the curve $y = 27 - x^2$ is

A. 108 B. 27 C. $12\sqrt{3}$ D. 3 E. NOTA

7. $\int_{\pi/6}^{\pi/2} \cot x \, dx =$

A. $\ln\frac{1}{2}$ B. $\ln 2$ C. $-\ln(2 - \sqrt{3})$ D. $\ln(\sqrt{3} - 1)$ E. NOTA

8. Find the slope of the curve $r = \cos 2\theta$ at $\theta = \frac{\pi}{6}$.

A. $\frac{\sqrt{3}}{7}$ B. $\frac{1}{\sqrt{3}}$ C. $\sqrt{3}$ D. 0 E. NOTA

9. The curve $x^3 + x \tan y = 27$ passes through (3, 0). Use local linearization to estimate the value of y at $x = 3.1$. The value is

A. –2.7 B. –0.9 C. 0 D. 0.1 E. NOTA

10. The coefficient of the x^2 term in the Taylor polynomial for $y = x^{2/3}$ around $x = 8$ is

A. $-\frac{1}{144}$ B. $-\frac{1}{72}$ C. $-\frac{1}{9}$ D. $\frac{1}{144}$ E. NOTA

11. The equation of the tangent to the curve $2x^2 - y^4 = 1$ at the point (-1, 1) is

A. $y = -x$ B. $y = 2 - x$ C. $4y + 5x + 1 = 0$

D. $x - 2y + 3 = 0$ E. NOTA

Calculus Individual – Mu Level
2000 Mu Alpha Theta National Convention

12. A curve is given parametrically by the equations $x = 3 - 2 \sin t$ and $y = 2 \cos t - 1$. The length of the arc from $t = 0$ to $t = \pi$ is

A. $\frac{\pi}{2}$ B. π C. $2 + \pi$ D. 2π E. NOTA

13. The only function that does not satisfy the Mean Value Theorem on the interval specified is

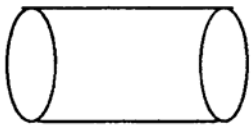
A. $f(x) = x^2 - 2x$ on [-3, 1] B. $f(x) = \frac{1}{x}$ on [1, 3] C. $f(x) = x^{2/3}$ on $\left[\frac{1}{2}, \frac{3}{2}\right]$

D. $f(x) = x + \frac{1}{x}$ on [-1, 1] E. NOTA

14. A cylindrical tank is partially full of water at time $t = 0$, when more water begins flowing in at a constant rate. The tank becomes half full when $t = 4$, and is completely full when $t = 12$. Let h represent the height of the water at time t. During which interval is $\frac{dh}{dt}$ increasing?

A. $0 < t < 4$ B. $0 < t < 8$ C. $0 < t < 12$

D. $4 < t < 12$ E. NOTA



15. $\int_1^2 \frac{dx}{\sqrt{4 - x^2}}$ is

A. $-\frac{\pi}{3}$ B. $\frac{\pi}{6}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{3}$ E. NOTA

16. The area inside $r = 3 \sin \theta$ and outside $r = 1 + \sin \theta$ is given by

A. $\int_{\pi/6}^{\pi/2} \left[9\sin^2\theta - (1 + \sin\theta)^2\right] d\theta$ B. $\int_{\pi/6}^{\pi/2} (2\sin\theta - 1)^2 \, d\theta$.

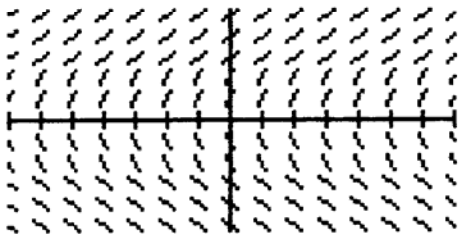
C. $\frac{1}{2}\int_{\pi/6}^{5\pi/6} (8\sin^2\theta - 1) d\theta$ D. $\frac{9\pi}{4} - \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1 + \sin\theta)^2 \, d\theta$ E. NOTA

Calculus Individual – Mu Level
2000 Mu Alpha Theta National Convention

17. $\lim_{x\to\infty} x^{1/x}$ is

A. 0 B. 1 C. e D. ∞ E. NOTA

18. Which equation has the slope field shown below?



A. $\frac{dy}{dx}=\frac{5}{y}$ B. $\frac{dy}{dx}=\frac{5}{x}$ C. $\frac{dy}{dx}=\frac{x}{y}$ D. $\frac{dy}{dx}=5y$ E. NOTA

19. Suppose the function f is both increasing and concave up on [a, b]. Then, using the same number of subdivisions, and with L, R, M, and T denoting respectively Left, Right, Midpoint, and Trapezoid sums, it follows that

A. $R \le T \le M \le L$ B. $R \le M \le T \le L$ C. $L \le T \le M \le R$

D. $L \le M \le T \le R$ E.. NOTA

20. Let $\int_0^x f(t)dt = x\sin\pi x$. Then $f(3)$ =

A. -3π B. -1 C. 1 D. 3π E. NOTA

21. If f '(x) = 2f (x) and f(2) = 1, then f(x) =

A. e^{2x-4} B. $e^{2x} + 1 - e^4$ C. e^{4-2x} D. e^{2x-1} E. NOTA

Calculus Individual – Mu Level
2000 Mu Alpha Theta National Convention

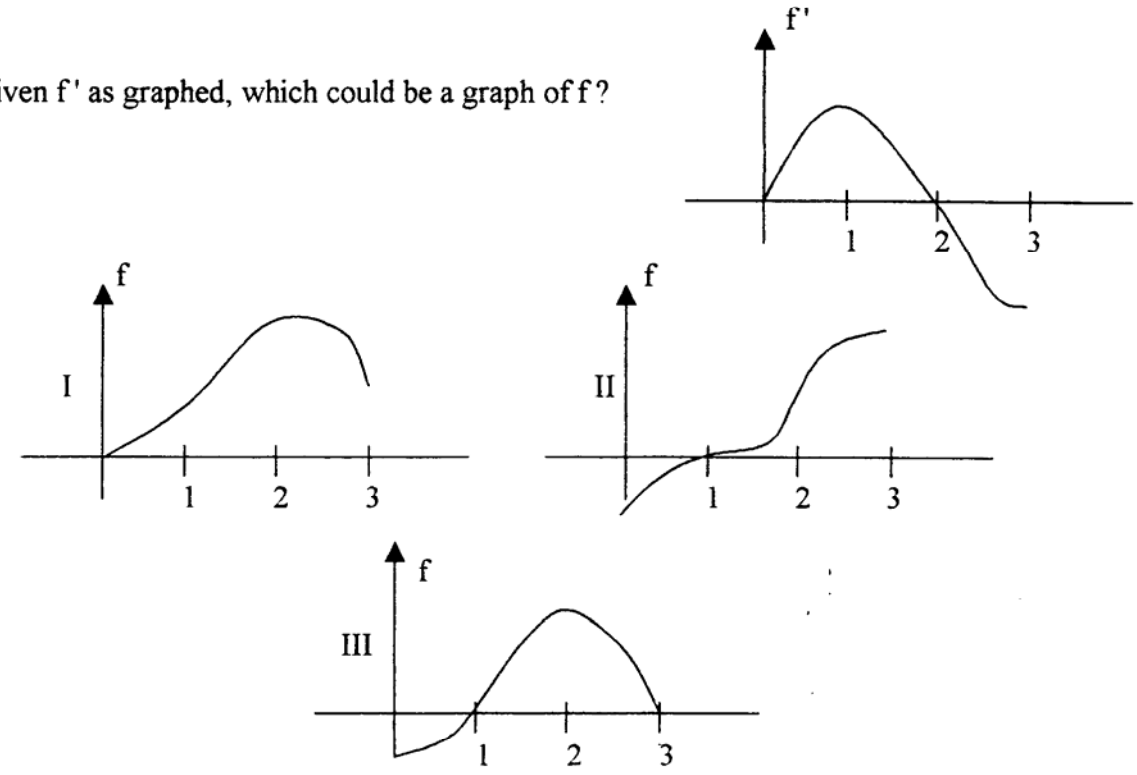
22. $\int_0^6 f(x-1)dx =$

A. $\int_{-1}^{7} f(x)dx$ B. $\int_{-1}^{5} f(x)dx$ C. $\int_{-1}^{7} f(x+1)dx$ D. $\int_{1}^{7} f(x)dx$ E. NOTA

23. Let $f(x) = x^5 + 1$ and let g be the inverse function of f. What is the value of g'(0)?

A -1 B. 1 C. $\frac{1}{5}$ D. g'(0) does not exist E. NOTA

24. Given f ' as graphed, which could be a graph of f ?



A. I only B. II only C. III only D. I and III E. NOTA

25. A particle moves on a straight line so that its velocity at time t is given by v = 4s, where s is its distance from the origin. If s = 3 when t = 0, then, when t = $\frac{1}{2}$, s equals

A. $1 + e^2$ B. $2e^3$ C. e^2 D. $2 + e^2$ E. NOTA

Calculus Individual – Mu Level
2000 Mu Alpha Theta National Convention

26. The first quadrant region bounded by $y = \frac{1}{\sqrt{x}}$, $y = 0$, $x = q$ $(0 < q < 1)$, and $x = 1$ is rotated about the x-axis. The volume obtained as $q \to 0^+$ equals

A. $\frac{2\pi}{3}$ B. $\frac{4\pi}{3}$ C. 2π D. 4π E. NOTA

27. The area of the surface generated by revolving the curve $y = \sin x$ from $x = 0$ to $x = \pi$ about the x-axis is

A. $2\pi\left(\sqrt{2} + \ln\left(1+\sqrt{2}\right)\right)$ B. $2\pi + \ln\left(1+\sqrt{2}\right)$ C. $2\pi + \frac{1}{2}\ln 2$

D. $2^{3/2}(\pi + \ln 2)$ E. NOTA

28. The interval of convergence of $\sum_{n=1}^{\infty} nx^n$ is

A. (-1, 1) B. [-1, 1) C. (-1, 1] D. [-1, 1] E. NOTA

29. Which of the following series converges?

A. $\sum \frac{1}{\sqrt[3]{n}}$ B. $\sum \frac{1}{\sqrt{n}}$ C. $\sum \frac{1}{n}$ D. $\sum \frac{1}{10n-1}$ E. NOTA

30. $\int \frac{x^2+2}{x^2+2x}dx =$

A. $\ln|x| + C$ B. $\ln|x| - 3\ln|x+2| + C$ C. $\frac{x^3}{3}\ln|x| + 2\ln|x+2| + C$

D. $\ln\left|\frac{x}{(x+2)^3}\right| + x + C$ E. NOTA