Mu Alpha Theta National Convention: Seattle, 1997
Calculus Individual Test

1. Determine the equation of the parabola with focus (4,3) and directrix x = -2.

(A) $x = \frac{1}{4}(y-3)^2 + 1$ (B) $y = \frac{1}{4}(x-1)^2 + 3$

(C) $x = \frac{1}{12}(y-3)^2 + 1$ (D) $y = \frac{1}{12}(x-1)^2 + 3$ (E) NOTA

2. What is the radius of the circle $x^2 + y^2 + 6x - 16y = 25$?

(A) 7 (B) 5 (C) $7\sqrt{2}$ (D) $5\sqrt{2}$ (E) NOTA

3. What is the eccentricity of $\frac{(x-3)^2}{64} + \frac{(y-1)^2}{36} = 1$?

(A) $\frac{5}{3}$ (B) $\frac{5}{4}$ (C) $\frac{\sqrt{7}}{3}$ (D) $\frac{\sqrt{7}}{4}$ (E) NOTA

4. The parametric equations of a particle's motion in the plane are $y = 1 + \sin t$ and $x = 1 - \cos t$. What is the shape of the path traced by the particle?

(A) an ellipse but not a circle (B) a parabola (C) a hyperbola (D) a circle (E) NOTA

5. What is the shape of the graph of $r = 1 + 3\cos\theta$?

(A) a circle (B) a limacon (C) a cardiod (D) a 4-leaf rose (E) NOTA

6. $g(x) = \frac{8+3x}{x^3}$ and $f(x) = x^2 g(x)$. Determine $f'(1)$.

(A) -8 (B) 6 (C) -4 (D) 2 (E) NOTA

7. A cube is expanding. When the edge length is increasing at 1 cm/s, the volume is increasing at a rate of 4 cm^3/s. What is the length of an edge at this moment?

(A) $\frac{\sqrt{2}}{2}$ (B) $\sqrt{2}$ (C) $\frac{2\sqrt{3}}{3}$ (D) $\frac{4\sqrt{3}}{3}$ (E) NOTA

8. Evaluate $\frac{\partial}{\partial x}\left(3xy+\frac{4y}{x}\right)$ at the point (3,2).

(A) $\frac{46}{9}$ (B) $\frac{62}{9}$ (C) 7 (D) $\frac{68}{9}$ (E) NOTA

9. What is the area of the region enclosed by the equation $r=\cos 3\theta$?

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{3}$ (E) NOTA

10. What is the volume of the solid formed by rotating the region between the graph of $y=4-(x-2)^2$ and the x-axis about the y-axis?

(A) $\frac{128\pi}{3}$ (B) 44π (C) $\frac{136\pi}{3}$ (D) $\frac{140\pi}{3}$ (E) NOTA

11. What is the average value of $y=x^3+4$ between 2 and 4?

(A) 31 (B) 34 (C) 37 (D) 40 (E) NOTA

12. If $f(b)=\int_5^b g(x)dx$ and $g(x)=\frac{2+3x}{x^2}$, what's $f'(4)$?

(A) 1 (B) $\frac{23}{24}$ (C) $\frac{11}{12}$ (D) $\frac{7}{8}$ (E) NOTA

13. Evaluate $\int\left(3+6x^2\right)dx$

(A) $3+18x^3$ (B) $3+2x^3$ (C) $3x+12x^3$ (D) $3x+2x^3$ (E) NOTA

14. Determine $y(4)$, given that $\frac{dy}{dx}=[x]$ and $y(6)=6$?

(A) -4 (B) -3 (C) -2 (D) -1 (E) NOTA

15. A particle undergoes an acceleration in the positive x direction which obeys the law $a=-2$. At time $t=2$ the particle is at $x=4$, while at time $t=1$ the particle's velocity is 6. Where is the particle at time $t=8$?

(A) -10 (B) -8 (C) -4 (D) 2 (E) NOTA

16. Determine the value of b given that f(x) is continuous for all x.

$$f(x)=\begin{cases} x+3, & x\le 8 \\ bx+8, & x>8 \end{cases}$$

(A) $\frac{1}{2}$ (B) $\frac{7}{16}$ (C) $\frac{3}{8}$ (D) $\frac{3}{16}$ (E) NOTA

17. What is the coefficient of the x^3 term in the Taylor expansion of $x+\frac{3}{(x-2)^2}$ about x = 0?

(A) $\frac{9}{2}$ (B) $\frac{3}{16}$ (C) $-\frac{3}{8}$ (D) (E) NOTA

18. Evaluate: $\lim_{x\to 4}\frac{3^{x-4}-1}{x^2-8x+16}$

(A) 0 (B) 1 (C) $\sqrt{3}$ (D) ∞ (E) NOTA

19. Evaluate: $\lim_{x\to 0}\frac{4x^2-25+\frac{9}{2x}}{2x-\frac{4}{x}-5}$

(A) 2 (B) $-\frac{9}{8}$ (C) $\frac{4}{3}$ (D) 5 (E) NOTA

20. In how many zeros does 100! end?

(A) 20 (B) 21 (C) 24 (D) 25 (E) NOTA

21. How many factors does 1750 have?

(A) 6 (B) 8 (C) 12 (D) 16 (E) NOTA

22. What is the value of a if $345_a = 456_{a-1}$?

(A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

23. For each n in the set of natural numbers from 3 to 10, the remainder when t is divided by n is 2. What is the smallest possible value for t?

(A) 2,522 (B) 54,382 (C) 1,814,402 (D) 3,628,802 (E) NOTA

24. A card is drawn from a deck. What is the probability that it is neither a 3 nor a club?

(A) $\frac{9}{13}$ (B) $\frac{17}{26}$ (C) $\frac{8}{13}$ (D) $\frac{31}{52}$ (E) NOTA

25. A class consists of 6 boys and 4 girls. If they line up with the boys in order by height and the girls randomly arranged in the same line, how many arrangements are possible?

(A) 3,656 (B) 4,132 (C) 4,488 (D) 5,040 (E) NOTA

26. Consider the data set: 3, 9, 7, 3, 11, 13, 4, x, y, where x equals the mode of the data set and y is the median of the data set. What is the maximum sum of x and y if both are integers?

(A) 10 (B) 12 (C) 14 (D) 18 (E) NOTA

27. You have two bags of coins. Bag 1 contains 5 quarters and 3 dimes. Bag 2 contains 3 quarters and 2 dimes. A coin is drawn from Bag 1 and placed in Bag 2. A pair of coins is then drawn from Bag 2. If the total amount of money drawn from Bag 2 is \$0.35, what is the probability a quarter was drawn from Bag 1?

(A) $\frac{8}{13}$ (B) $\frac{27}{39}$ (C) $\frac{33}{49}$ (D) $\frac{40}{67}$ (E) NOTA

28. What is the expected value of number Q, if the probability that Q is less than x is $p_Q(x) = \left(\frac{x}{3}\right)^2$ for $0 \le x \le 3$, is 0 for $x < 0$, and 1 for $x > 3$.

(A) $\frac{79}{36}$ (B) $\frac{9}{4}$ (C) $\frac{41}{18}$ (D) $\frac{5}{2}$ (E) NOTA

29. The sum of the first 86 terms of an arithmetic sequence with common difference -3 is 215. What is the 30th term of the sequence?

(A) 40 (B) 43 (C) 46 (D) 49 (E) NOTA

30. Evaluate: $\sum_{n=15}^{27} n^3 - an$

(A) $122{,}139 - 273a$ (B) $128{,}390 - 273a$
(C) $131{,}859 - 273a$ (D) $138{,}242 - 273a$ (E) NOTA

31. The first, third, and fourth terms of a geometric sequence form an arithmetic sequence. If the sequences are non-trivial and the common ratio of the geometric sequence is positive, what is the common ratio?

(A) $\frac{-1+\sqrt{5}}{2}$ (B) $-1+\frac{\sqrt{5}}{2}$ (C) $\frac{1+\sqrt{5}}{2}$ (D) $1+\frac{\sqrt{5}}{2}$ (E) NOTA

32. A rubber ball dropped 40 meters bounces $\frac{3}{8}$ of the height from which it fell on each bounce. How far, in meters, will it travel before coming to rest?

(A) 64 (B) 72 (C) 80 (D) 88 (E) NOTA

33. Find the sum of all the multiples of 27 between 432 and 2345, inclusive.

(A) 89,245 (B) 97,767 (C) 105,388 (D) 111,206 (E) NOTA

34. What is the period of $y = 3\sec(24\pi x)$?

(A) $\frac{1}{24\pi}$ (B) $\frac{1}{12\pi}$ (C) $\frac{1}{24}$ (D) $\frac{1}{12}$ (E) NOTA

35. Which of the following statements is equivalent to $\cos^2 2x$?

(A) $1 - 2\sin^2 x$ (B) $1 - \sin^3 x$ (C) $2\cos^2 x - 1$ (D) $\frac{1+\cos 4x}{2}$ (E) NOTA

36. If $\tan x = \frac{1}{2}$, and x is between $\frac{\pi}{2}$ and π, what is the value of $\cos 2x$?

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$ (C) $\frac{3}{5}$ (D) $\frac{3}{4}$ (E) NOTA

37. A particle is constrained to always occupy a point on the unit circle and in the first quadrant. Let θ be the angle from the positive x-axis to the ray through the origin containing the particle under consideration. When the particle's movement satisfies $y = .3$ units, $\frac{dy}{dt} = -.6$ units/second, what is $\frac{d\theta}{dt}$ at this time, to the nearest hundredth of a rad/sec?

(A) .47 (B) .52 (C) .59 (D) .63 (E) NOTA

38. $y = \csc^{-1} x$. Determine $\frac{dy}{dx}$.

(A) $\frac{1}{x\sqrt{x^2+1}}$ (B) $\frac{1}{x\sqrt{x^2-1}}$ (C) $\frac{1}{\sqrt{x^2+1}}$ (D) $\frac{1}{\sqrt{x^2-1}}$ (E) NOTA

39. In a sequence of four numbers, the first three form an arithmetic sequence, while the last three form a geometric sequence. If the common difference of the arithmetic sequence and the common ratio of the geometric sequence are equal, and the second of the four terms is $\frac{7}{5}$, what is the value of the common ratio of the geometric sequence?

(A) $\frac{7}{2}$ (B) $\frac{7}{5}$ (C) $\frac{2}{5}$ (D) $\frac{5}{7}$ (E) NOTA

40. Which of the following have continuous first derivatives for all values of $x \in \Re$?

I. $y = x$
II. $y = |x|$
III. $y = x^2$
IV. $y = [x]$
V. $y = -x$

(A) I, II & III (B) II, III & IV (C) I, III & V (D) I, II, III & V (E) NOTA