

Mu Alpha Theta Convention 2004
MU APPLICATIONS

For each question, NOTA means “none of the above is correct.”

1. For $f(x) = 2x + 1$, $g(x) = x^2$, and $h(x) = 1 - x$, find $f \circ g \circ h(2)$.
(A) 3 (B) 2 (C) 1 (D) 0 (E) NOTA

2. Evaluate $\lim_{x \rightarrow 2} \frac{\sqrt{6-x} - 2}{x-2}$.
(A) $-\frac{1}{4}$ (B) 0 (C) $\frac{1}{4}$ (D) 4 (E) NOTA

3. Find the equation of the tangent to the curve $y = 3x - 2x^2$ through the point (1, 1).
(A) $y = -x + 2$ (B) $y = \frac{4}{3}x - \frac{1}{3}$ (C) $y = x$ (D) $y = 2x - 1$ (E) NOTA

4. Evaluate $f'(2\pi/3)$ for the function $f(x) = (\cos 2x)^3$,
(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{9}{8}$ (C) $\frac{3}{4}$ (D) $-\frac{3\sqrt{3}}{4}$ (E) NOTA

5. Evaluate the definite integral $\int_1^2 \frac{x}{x^2+1} dx$.
(A) $\ln \frac{\sqrt{10}}{2}$ (B) $\arctan 2 - \frac{\pi}{4}$ (C) $\ln \frac{5}{2}$ (D) $\arctan 2$ (E) NOTA

6. Find dy/dx for the equation $xy = \tan(xy)$.
(A) $\sec^2(xy) + 1$ (B) $\sec^2(xy) - 1$ (C) $-y/x^2$ (D) $-y/x$ (E) NOTA

7. The position function for a particle moving along the x-axis for time $t > 0$ is $s(t) = 8 + 2t - t^2$. Find the speed and direction of the particle when $t = 5$.
(A) 7, left (B) 8, left (C) 7, right (D) 8, right (E) NOTA

8. Find the area under the curve $f(x) = x \cos x$ for $0 \leq x \leq \frac{\pi}{2}$.
(A) $\frac{\pi}{2} - 1$ (B) $\frac{\pi}{2}$ (C) $\pi - 1$ (D) $\frac{\pi - 1}{2}$ (E) NOTA

9. Solve the differential equation $y' = y^2 + 1$ with the initial condition $y(1) = 0$.
- (A) $y = \tan(x - 1)$ (B) $y = x - 1$ (C) $y = \ln x$ (D) $y = e^{x-1} - 1$ (E) NOTA
10. Find the area of the region enclosed by the curve $r = 3\cos\theta$.
- (A) 9π (B) $\frac{9\pi}{2}$ (C) 3π (D) $\frac{9\pi}{4}$ (E) NOTA
11. Find the angle between the tangents through the points of intersection of the parabola $y^2 = 4x$ and the circle $x^2 + y^2 + 2x = 7$.
- (A) 53.7° (B) 90° (C) 88.2° (D) 60° (E) NOTA
12. Find the limit of the sequence $\{\sqrt{x}, \sqrt{x\sqrt{x}}, \sqrt{x\sqrt{x\sqrt{x}}}, \dots\}$ for $x = 4$.
- (A) 2 (B) $2\sqrt{2}$ (C) 4 (D) 8 (E) NOTA
13. The path of a particle traveling in the xy -plane given by the parametric equations $x(t) = t + 2$ and $y(t) = t^2 + 4$. Find dy/dx when $x = 3$.
- (A) -2 (B) $\frac{1}{2}$ (C) 2 (D) 6 (E) NOTA
14. Let $P(x)$ be the profit (in dollars) from manufacturing and selling x luxury cars where $P(100) = 90,000$ and $P'(100) = 1200$. Estimate the value of $P(99)$.
- (A) 120,000 (B) 118,800 (C) 99,000 (D) 92,000 (E) NOTA
15. Describe the family of curves that are orthogonal to the family of curves $x = ky^2$, where k is an arbitrary constant.
- (A) circles (B) ellipses (C) parabolas (D) hyperbolas (E) NOTA
16. For $2 \leq x \leq 4$, find the area of the region between the curves $y = x^3 - 3x^2$ and $y = 0$.
- (A) $\frac{13}{4}$ (B) 4 (C) $\frac{23}{4}$ (D) $\frac{19}{2}$ (E) NOTA

17. Given that the first five answers of a 30 question, 5-option, multiple choice test are not identical, what is the probability that exactly one of the first five is different from the other four?
- (A) $\frac{1}{3125}$ (B) $\frac{1}{625}$ (C) $\frac{4}{125}$ (D) 1 (E) NOTA
18. Find the maximum slope of the curve $y = \frac{e^x}{1+e^x}$?
- (A) $\frac{1}{4}$ (B) $\frac{1}{e}$ (C) $\frac{1}{2}$ (D) 1 (E) NOTA
19. A toy rocket fired straight up into the air has height of $s(t) = 160t - 16t^2$ feet after t seconds. What is the average velocity (feet per second) of the rocket from take-off until it reaches its maximum height?
- (A) 25 (B) 40 (C) 50 (D) 80 (E) NOTA
20. Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side 8 if one side of the rectangle lies on the base of the triangle.
- (A) $6, \frac{4\sqrt{3}}{3}$ (B) $4, 2\sqrt{3}$ (C) $2, 4\sqrt{3}$ (D) $2\sqrt{2}, 2\sqrt{6}$ (E) NOTA
21. A news item is spread by word of mouth to a potential audience of 10,000 people. After t days, $f(t) = 10,000 / (1 + 50e^{-4t})$ people will have heard the news. In how many days (rounded to three decimal places) will the news be spreading at the greatest rate?
- (A) .556 (B) .724 (C) .978 (D) 1.223 (E) NOTA
22. A ladder 25 feet long rests against a house. A man takes hold of the bottom of the ladder and walks off with it with a uniform velocity of 2 feet a second. How fast (feet per second) is the top of the ladder descending when the bottom is 7 feet from the house?
- (A) $\frac{5}{12}$ (B) $\frac{7}{12}$ (C) $\frac{17}{12}$ (D) 2 (E) NOTA
23. For the given parametric equations $x = 4/t$ and $y = 2t$, find d^2y/dx^2 in terms of the given parameter.
- (A) $\frac{2}{t^2}$ (B) 0 (C) $-\frac{t}{2}$ (D) $\frac{-t^2}{4}$ (E) NOTA

24. Find the sum, if it exists, of the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{12} + \dots$ where the terms are the reciprocals of the positive integers whose only prime factors are 2's and 3's.
- (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 6 (E) NOTA
25. Beginning at $t = 0$ a particle travels along the x -axis with velocity $v(t) = 2t - t^2$. Approximate to three decimal places the amount of time it will take for the particle to return to its original position.
- (A) 1.333 (B) 2.667 (C) 2.833 (D) 3.000 (E) NOTA
26. The ends of a trough 6 feet long are isosceles right triangles having the hypotenuse horizontal. A horse is drinking the water at the rate of a cubic foot per minute. How fast (inches per minute) is the level of the water sinking when the water is 18 inches deep?
- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{4}{3}$ (E) NOTA
27. For the curve $y = x^2 - \frac{8}{x}$, how many times does the concavity change its direction?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA
28. Suppose the average time spent on a problem of this topic test is two minutes. Find the probability (rounded to three decimal places) that a student spends between 2 and 3 minutes on this problem given that the probability density function is
- $$f(t) = \begin{cases} 0 & \text{if } t < 0 \\ 0.5e^{-t/2} & \text{if } t \geq 0 \end{cases}.$$
- (A) 0.148 (B) 0.136 (C) 0.112 (D) 0.072 (E) NOTA
29. The position of one particle at time t is given by $x_1 = 3\sin t$ and $y_1 = 2\cos t$. The position of a second particle is given by $x_2 = -3 + \cos t$ and $y_2 = 1 + \sin t$. For $t > 0$ at approximately what time do the two particles collide?
- (A) 0.785 (B) 1.871 (C) 3.142 (D) 4.712 (E) NOTA
30. Find the maximum area of a rectangle that can be circumscribed about a given rectangle with length 3 and width 5.
- (A) 15 (B) 17 (C) 30 (D) 32 (E) NOTA

Tie Breakers

1. What is the 20th term of the Fibonacci sequence 1, 1, 2, 3, 5, 8...?
2. What is the 20th triangular number?
3. Find the number of distinguishable permutations that can be made from the letters in the word HAWAII?