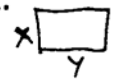
SOLUTIONS – MU LEVEL
2000 MU ALPHA THETA NATIONAL CONVENTION

1.

$\int dx = x + C$

D

2.



$2x+2y=20$

$x+y=10$

$\pi y^2 x = V$

$\pi(y^2)(10-y)=V$

max @ $y = 6\frac{2}{3}$

$x + 6\frac{2}{3} = 10$

$x = 3\frac{1}{3}$

$A = xy$

$A = (3\frac{1}{3})(6\frac{2}{3})$

$A \approx 22$

B

3.

$y = x^{x+1}$

$y' = x^{x+1}\ln x + (x+1)x^x$

$y' = (x\ln x + x + 1)(x^x)$

A

4.

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$

$\sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} \cdots$

so, coeff of $x^{14} = -\frac{1}{7!}$

A

5. A of polar grph $= \int r^2 \, d\theta$

$A = \left(\frac{1}{2}\int_0^{\pi/2} 9\sin^2\theta\cos^2\theta \, d\theta\right)(4)$

$= 2\int_0^{\pi/2} 9\sin^2\theta\cos^2\theta \, d\theta = 9\pi/8$

D

6. Avg val. $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{3}\int_0^3 \frac{1}{\sqrt{x^2+1}}\,dx$

$\frac{1}{3}\left(\ln\left(x+\sqrt{x^2+1}\right)\Big|_0^3\right)$

$\frac{1}{3}\left(\ln(3+\sqrt{10})\right)$

B

7. Simpson's Method

$n=4$ $f(x)=\frac{1}{x}$ $\Delta x = .5$

$\frac{.5}{3}\left(f(7)+4f(7.5)+2(f(8)) + 4f(8.5)+f(9)\right) =$

$\frac{1}{6}\left(\frac{1}{7}+\frac{4}{7.5}+\frac{1}{4}+\frac{4}{8.5}+\frac{1}{9}\right) \approx .2513$

B

8.

$V = \pi\int_0^h y\,dy$

$V = \frac{\pi h^2}{2}$

$\frac{dV}{dt} = \pi h \frac{dh}{dt}$

$1 = \pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{\pi}$

A

9.

$y = \sqrt[3]{x} = x^{\frac{1}{3}}$

$dy = \frac{1}{3}x^{-\frac{2}{3}}\,dx$

$dy = \frac{1}{3}(27)^{-\frac{2}{3}}\ (29-27)$

$dy = \frac{1}{3\cdot 9}(2) = \frac{2}{27} \approx .074$

D

10.

$\frac{dy}{dx} = xy^2$

$\frac{1}{y^2}dy = x\,dx$

$-\frac{1}{y} = .5x^2 + C$

insert $(0,4)$ $-\frac{1}{4} = 0 + C$

$C = -.25$

$-\frac{1}{y} = \frac{1}{2}x^2 - \frac{1}{4}$

$-\frac{1}{y} = \frac{2x^2-1}{4}$

$y = \frac{4}{1-2x^2}$

C

11.

$x = \sqrt{t^2+3}$ $y = \sin t^2$

$x^2 = t^2+3$

$x^2-3 = t^2$

$y = \sin(x^2-3)$

$\frac{dy}{dx} = 2x\cos(x^2-3)$

$\frac{d^2y}{dx^2} = 2\cos(x^2-3) - 4x^2\sin(x^2-3)$

E

12 Pappus Rule

$\int_1^4 \frac{1}{\sqrt{x}}\,dx = \text{area} = 2$

$\frac{\int_1^4 \frac{1}{\sqrt{x}}\cdot x\,dx}{2} = \bar{x} = \frac{7}{3}$

$\frac{\int_1^4 \left(\frac{1}{\sqrt{x}}\right)^2 dx}{2\cdot 2} = \frac{\ln 2}{2} = \bar{y}$

cent $= \left(\frac{7}{3}, \frac{\ln 2}{2}\right)$

$y = x-7$ $m=1$ $c=-7$

volume $= 2\pi r \cdot$ mass

$r = \frac{\left|\frac{7}{3}\cdot 1 - \frac{\ln 2}{2} - 7\right|}{\sqrt{1^2+1}} = \approx 3.544896$

$V = 2\pi r \cdot \text{mass}$

$V \approx 44.5465$

tenths digit $= 5$

B

13.

$V = \frac{4}{3}\pi r^3$

$.05 = \frac{4}{3}(3)(\pi)(9)\left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = \frac{.05}{36\pi}$

$SA = 4\pi r^2$

$\frac{dSA}{dt} = 8\pi r\frac{dr}{dt}$

$= 8\pi(3)\left(\frac{.05}{36\pi}\right)$

$= \frac{1}{3}(.2) = \frac{1}{15}$

$r=3$

C

14.

MVT $= f'(c) = \frac{f(b)-f(a)}{b-a}$

$f'(c) = f(1) - f(0)$

$f'(c) = \frac{\tan^{-1} 1}{2}$

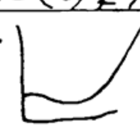
$f'(c) = \frac{\pi}{8}$

$f'(x) = \frac{1}{x^2+1} - \frac{2x\tan^{-1}x}{(x^2+1)^2}$

$\frac{1 - 2x\tan^{-1}x}{(x^2+1)^2} = \frac{\pi}{8}$ $x \approx .471$

C

15.



Trapezoidal Rule

$A = \frac{1}{2}(2)\left(f(0)+2f(2)+2f(4)+f(6)\right)$

$A = 4 + 2(4) + 2(36) + 148 =$

232

C

16. curve has slope of reciprocal of orig. curve

$\frac{2}{x^3}$ $y'(1) = -2$

$y_2' = -\frac{1}{-2} = \frac{1}{2}$ $y = \frac{1}{2}x + \frac{1}{2}$

$y = \frac{1}{2}x + b$ $2y - x = 1$

$1 = \frac{1}{2} + b$

$b = \frac{1}{2}$

D

17. def of derivative @ $x = 6$.

$f(x) = \ln(\ln x)$

$f'(x) = \frac{1}{x \ln x}$

$f'(6) = \frac{1}{6 \ln 6}$

C

18 max of $f(x)$ where $f'(x) = 0$ & $f''(x) < 0$

max occurs @ $x = \frac{1}{\sqrt{2}}$

$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} e^{-\left(\frac{1}{\sqrt{2}}\right)^2}$

$= \frac{1}{\sqrt{2}} \cdot e^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{e}} = \frac{1}{\sqrt{2e}}$

C

19. Area of semicircle $= \frac{1}{2}\pi r^2$

$r = \frac{1}{2}(\cos x - \sin x)$

$A = \int_0^{\pi/4} \pi \left(\frac{1}{2}(\cos x - \sin x)\right)^2 dx$

$\frac{\pi}{4} \int_0^{\pi/4} (\cos x - \sin x)^2 dx \approx \ldots$

A

20. $h(x) = \int_0^{x^3} \left(t^4 \sin^2 t^2 + \frac{1}{2} t^3\right) dt$

$h'(x) = x^{12} \, 3x^2 \sin^2 x^6 + \frac{3}{2} x^{11}$

$= 3x^{14} \sin^2 x^6 + \frac{3}{2} x^{11}$

A

21. $2x + 2yy' = \cos(xy)(y + xy')$

$2x + 2yy' = y\cos(xy) + xy'(\cos(xy))$

$y'(2y - x\cos(xy)) = y\cos(xy) - 2x$

$y' = \frac{y\cos(xy) - 2x}{2y - x\cos(xy)}$

D

22. $v = \ln(\ln(\ln x))$

$dv = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}\, dx$

$\int \frac{dv}{v} = \ln v$

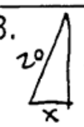
$= \ln(\ln(\ln(\ln x))) \Big]_{e^e}^{e^{e^2}}$

$= \ln 2 - \ln 1$

$= \ln 2$

B

23.



$\downarrow .1$ ft/sec $5^2 + x^2 = 20^2$

$x = 5\sqrt{15}$

$x^2 + y^2 = 20^2$

$2x\,dx + 2y\,dy = 0$

$2(5\sqrt{15})dx + 10(.1) = 0$

$dx(10\sqrt{15}) = -1$

$dx = \frac{-1}{10\sqrt{15}} = \frac{-\sqrt{15}}{150}$

C

24. $\ln \sqrt{x^2 - 1} > 0$

$\sqrt{x^2 - 1} > 1$

$x^2 - 1 > 1$

$x^2 > 2$ $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$|x| > \sqrt{2}$

$x > \sqrt{2}$

$x < -\sqrt{2}$

C

25. $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{k}}$

$6 = 180 \left(\frac{1}{2}\right)^{\frac{t}{1800}}$

$\frac{1}{30} = \left(\frac{1}{2}\right)^{t/1800}$

$-\ln 30 = -t/1800 \ln 2$

$\frac{\ln 30}{\ln 2}(1800) = t$

$t \approx 8832.40$

C

26. $\sin t = t - \frac{1}{6}t^3 + \frac{1}{120}t^5$

$\frac{\sin t}{t} = 1 - \frac{1}{6}t^2 + \frac{1}{120}t^4$

$\int_0^x \frac{\sin t}{t}\, dt = x - \frac{1}{18}x^3 + \frac{1}{600}x^5$

B

27. $\frac{dv}{dt} = -40$ $SA = 6s^2$

$V = s^3$ $\frac{dSA}{dt} = 12s\frac{ds}{dt}$

$\frac{dv}{dt} = 3s^2 \frac{ds}{dt}$ $= 12(5)\left(-\frac{8}{15}\right)$

$-40 = 3(25)\frac{ds}{dt}$ $= -32.50$

$\frac{ds}{dt} = -\frac{8}{15}$ decreasing @ $32 \frac{cm^2}{sec}$

D

28. $j = -8t = a'$

$a = \int -8t\, dt = -4t^2 + c$

$0 = -4(3^2) + c$

$36 = c$

$a = -4t^2 + 36$

$v = a'$

$v = \int -4t^2 + 36\, dt$

$v = -\frac{4}{3}t^3 + 36t + c$

$0 = -\frac{4}{3}(27) + 108 + c$

$c = -72$

$\boxed{v(t) = -\frac{4t^3}{3} + 36t - 72}$

A

29. $400 = 150t$

$t = \frac{8}{3}$

$s\left(\frac{8}{3}\right) = 3 + 53\left(\frac{8}{3}\right) - 16\left(\frac{8}{3}\right)^2$

$= \frac{275}{9}$

$\frac{275}{9} - 9 = \frac{194}{9}$

B

30. definition of derivative

$\lim_{x \to 3} \frac{g(x) - g(3)}{x - 3} = 1.743$

this shows deriv is pos @ $x = 3$ so graph is increasing

B