B (18)

1989 NATIONAL MAΘ CALCULUS TEST

1) If $[[x]]$ is the greatest integer function of x, find the nth derivative of $f(x)=\left[x-[[x]]\right]^n$ evaluated at x=2.5.

a) 0 b) n! c) (n-1)! d) (n+1)! e) nota

2) Evaluate: $\sum_{n=0}^{\infty} \frac{1}{n^2+5n+6}$

a) 0 b) 1/6 c) 1/3 d) 1/2 e) nota

3) If $f'(x) = \frac{f(x)}{1+x^2}$ and $f(1) = 2e^{\pi/4}$, find $\lim_{x\to\infty} f(x)$.

a) $e^{\pi/2}$ b) $e^{\pi/4}$ c) $e^{3\pi/4}$ d) $2e^{\pi/4}$ e) nota

4) The functions f and g are defined for all positive real numbers such that: $f(x^2)=x^3$ and $g'(x^2)=x^3$ and $g(1)=0$.

Find: $f'(4)$

a) 3 b) 8 c) 12 d) 16 e) nota

5) The slope of a curve at (x,y) is $\frac{dy}{dx} = (x-2)(x-3)$.
At the point where x=2, the curve has a:

a) vertical tangent b) point of inflection c) maximum
d) minimum e) nota

6) A function is defined parametrically by $x=f(t^2)$ and $y=g(t)$, then $\frac{dy}{dx}$ is:

a) $\frac{g'(t)}{f'(t)}$ b) $\frac{g'(t)}{(t^2)\ f'(t)}$ c) $\frac{g'(t)}{(2t)\ f'(t)}$

d) $\frac{g'(t)}{(2t)f'(t^2)}$ e) nota

7) $\frac{d}{dx}(\ln e^{2x}) = ?$

a) $\frac{1}{e^{2x}}$ b) $\frac{2}{e^{2x}}$ c) 2x d) 2 e) nota

8) Let f and g be differentiable ~~fractions~~ functions such that

$f(1)=2$, $f'(1)=3$, $f'(2)=-4$

$g(1)=2$, $g'(1)=-3$, $g'(2)=5$

If $h(x)=f(g(x))$, then $h'(1)=?$

a) -9 b) -4 c) 0 d) 12 e) nota

9) If $f(x)=\frac{x^3}{3} - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \leq x \leq 9$, then the absolute maximum value of the function occurs when x is:

a) 0 b) 2 c) 6 d) 9 e) nota

10) The region in the first quadrant bounded by the graph of $y=\sec(x)$, $x=\pi/4$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

a) $\frac{\pi^2}{4}$ b) $\pi-1$ c) π d) 2π e) nota

11) If the line $3x-4y=0$ is tangent in the first quadrant to the curve $y=x^3+k$, then k is?

a) 1/2 b) 1/4 c) 0 d) -1/8 e) nota

12) If $\frac{x+1}{x^3+x^2-6x}$ is decomposed into partial fractions, then the coefficient of $\frac{1}{x-3}$ is ?

a) 0 b) -1/6 c) 3/10 d) -2/15 e) nota

13) If $f(x)=(x^2+1)^{(2-3x)}$, then $f'(1)=?$

a) $(-1/2)\ln 8e$ b) $-\ln 8e$ c) $(-3/2)\ln 2$ d) -1/2 e) nota

14) If $f''(x)-f'(x)-2f(x)=0$, $f'(0)=2$, and $f(0)=2$, then $f(1)=?$

a) e^2+e^{-1} b) 1 c) 0 d) e^2 e) nota

15) What are the coordinates of the inflection point on the graph of $y=(x+1)\ \text{Arctan}\ (x)$?

a) $(-1,0)$ b) $(0,0)$ c) $(0,1)$ d) $(1,\pi/2)$ e) nota

16) The area of the closed region bounded by the polar graph of $r=\sqrt{3+\cos\theta}$ is given by the integral:

a) $\int_0^{\pi}(3+\cos\theta)\,d\theta$ b) $\int_0^{\pi}\sqrt{3+\cos\theta}\,d\theta$

c) $\int_0^{\pi/2}(3+\cos\theta)\,d\theta$ d) $\int_0^{\pi/2}\sqrt{3+\cos\theta}\,d\theta$ e) nota

17) At $x=0$, which of the following is true of the function defined by $f(x)=x^2+e^{-2x}$?

a) f is increasing b) f is decreasing c) f is discontinuous
d) f has a relative maximum d) nota

18) If $f(x)=\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$, for $x\neq 2$, and $f(2)=k$

If f is continuous at $x=2$, then $k=$?

a) 0 b) 1/6 c) 1/3 d) 1 e) nota

19) Which of the following can have a Maclaurin series?

a) $|x|$ b) $\ln(x)$ c) $\sqrt{x}$ d) $\cot(x)$ e) nota

20) The derivative of $(1+x)^n \ln(1+x)$ with respect to $(1+x)$ is:

a) $(1+x)^{n-1}\left(1+n \ln (1+x)\right)$ b) $1+ \ln (1+x)$ c) $n \ln (1+x)^n$

d) $\frac{1}{n} + \ln (1+x)$ e) nota

21) A spotlight is situated 40' from a straight wall. The light is rotated at 5 degrees/second about a vertical pole. The rate at which the beam moves along the wall when it is inclined 30° to the line of shortest distance is:

a) $\frac{60}{31}\pi$ fps b) $\frac{40}{9}\pi$ fps c) $\frac{40}{27}\pi$ fps d) $\frac{10}{3}\pi$ fps e) nota

22) Let $f(x)$ be a differentable function of x and let $\frac{d\left(f(x)\right)}{dx} = g(x)$

If $y=\left(1+f(x)\right)2$, then $\frac{dy}{dx} = ?$

a) $2\left(1+g(x)\right)$ b) $2\left(1+f(x)\right) \cdot g(x)$ c) $2\left(1+g(x)\right) \cdot f(x)$

d) $2\left(1+f(x)\right) \cdot f(x) \cdot g(x)$ e) nota

23) The position of a particle $P(x,y)$ at time t is given by $x=\cos t$, $y=\sin t$. The distance travelled between $t=0$ and $t=2\pi$ is:

a) $\pi/2$ b) π c) 2π d) 4π e) nota

24) If $y=x^x$ and where $x>0$, then a turning point of the curve occurs where $x = ?$

a) $1/e$ b) e c) $1+e$ d) $1-e$ e) nota

25) A Taylor series expansion of $f(x) = \frac{1}{x}$ about a=-1 is:

a) $\sum_{k=0}^{\infty} \frac{(x+1)^k}{(-1)^k}$ b) $\sum_{k=0}^{\infty} (x+1)^k$ c) $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k!}$

d) $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k!}$ e) nota

26) The mean value theorem guarantees the existence of a special point on the graph of $y=\sqrt{x}$ between (0,0) and (4,2). What are the coordinates of this point?

a) (2,1) b) $(2,\sqrt{2})$ c) (1,1) d) $(\frac{1}{2}, \frac{1}{\sqrt{2}})$ e) nota

27) The reasonable two-place approximation for $\sqrt[4]{84}$ found by the use of differentials is:

a) 2.98 b) 3.01 c) 3.03 d) 3.05 e) nota

28) A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t-3t^2$. What is the <u>total</u> distance covered by the point between t=1 and t=2 ?

a) 1 b) 4/3 c) 5/3 d) 2 e) nota

29) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ II. $\sum_{n=1}^{\infty} \frac{1}{n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

a) I only b) III only c) I and II only d) I and III only
e) I, II and III

30) If f is a continuous function on [a,b], which of the following is necessarily true?

a) f' exists on (a,b).

b) If $f(x_o)$ is a maximum of f, then $f'(x_o) = 0$.

c) $\lim_{x \to x_o} f(x) = f(\lim_{x \to x_o} x)$, for $x_o \in (a,b)$.

d) $f'(x) = 0$ for some $x \in [a,b]$.

e) nota