Trigonometry Answer Sheet:

1. A
2. A
3. B
4. C
5. C
6. D
7. B
8. E
9. E
10. E
11. A
12. C
13. B
14. E
15. A
16. C
17. A
18. B
19. A
20. B
21. B
22. A
23. A
24. B
25. D

Trigonometry Topic Test
Solutions

① $\cot \frac{1}{2}x = \dfrac{1+\cos x}{\sin x}$ A

$= \dfrac{1+\frac{4}{5}}{-\frac{3}{5}}$

$= \dfrac{+9}{-3} = -3$

② By inspection Amplitude $= 4$ A

③ $1 + \tan^2 x = \csc^2 56°$ B

$\sec^2 x = \csc^2 56°$

$\sec x = \csc 56°$

$\therefore x = 34°$

④ $\dfrac{1}{\cos^2 x} + \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x} = \dfrac{1+\sin x}{\cos^2 x}$ C

$= \dfrac{\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}(1-\sin x)}$

$= \dfrac{1}{1-\sin x}$

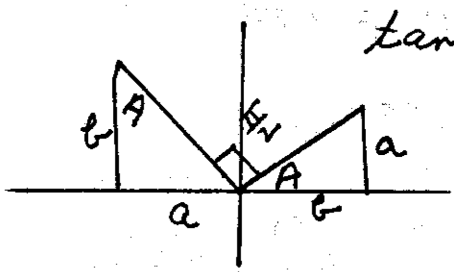
⑤ $\sin\frac{3x}{5}\cos\frac{2x}{5} + \cos\frac{3x}{5}\sin\frac{2x}{5} = \frac{1}{2}$ C

$\sin\left(\frac{3x}{5}+\frac{2x}{5}\right) = \frac{1}{2}$

$\sin x = \frac{1}{2}$

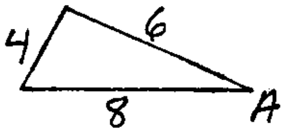
⑥ $\tan\left(\frac{3\pi}{2}+A\right) = \tan\left(\frac{\pi}{2}+A\right)$ D

$\tan A = \frac{a}{b}$



$\tan\left(\frac{\pi}{2}+A\right) = -\frac{b}{a}$

$\tan\left(\frac{\pi}{2}+A\right) = -\cot A$

(7) 

$16 = 36 + 64 - 96\cos A$

$96\cos A = 84$

$\cos A = \frac{7}{8}$

B

(8) $\cos x(2\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

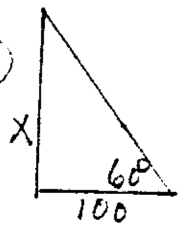
$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

Sum: $\frac{\pi}{2} + \frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = 3\pi$

E

(9) $\frac{\cancel{(\sin B + \cos B)}(\sin^2 B - \sin B\cos B + \cos^2 B)}{\cancel{(\sin B + \cos B)}}$

E

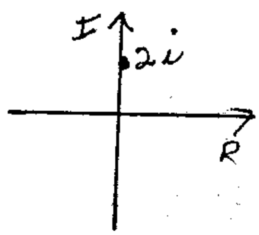
(10) 

$\tan 60° = \frac{x}{100}$

$x = 100 \cdot \sqrt{3}$

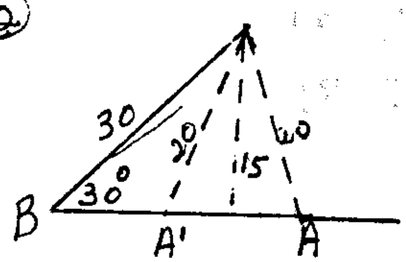
$x = 173.2$

E

(11) 

$2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

A

(12) 

C

2 solutions $b > a \sin b$

$\frac{\sin A}{30} = \frac{\sin B}{20}$

$\sin A = (30 \cdot \frac{1}{2}) \div 20 = \frac{3}{4}$

(13) $\frac{\cancel{150}^{\,5}}{\cancel{360}_{\,6}}(\cancel{50}^{\,25}\pi) = \frac{125\pi}{6}$

B

(14) $\text{Arctan}\ \frac{1}{2}x + \text{Arctan}\ \frac{2}{3}x = \frac{\pi}{4}$

$\tan(A+B) = \tan\frac{\pi}{4}$ E

$\frac{\frac{1}{2}x + \frac{2}{3}x}{1 - \frac{1}{3}x^2} = 1$

$7x = 6 - 2x^2$

$2x^2 + 7x - 6 = 0$

$x = \frac{-7 \pm \sqrt{97}}{4}$

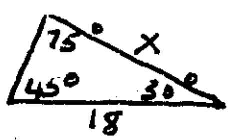
but since we want principal solution only 1 solution

A

(15) $\csc(\sin^{-1}\frac{1}{2} - \cos^{-1}\frac{1}{2})$

$\csc(30^\circ - 60^\circ) = -\csc 30^\circ$

$= -2$

(16) 

$\text{area} = \frac{1}{2} \cdot 18 \cdot x \cdot \sin 30^\circ$ C

$\frac{x}{\frac{\sqrt{2}}{2}} = \frac{18}{\sin 75^\circ}$ $\quad = \frac{1}{2}\,18(18\sqrt{3}-18)\cdot\frac{1}{2}$

$\frac{2x}{\sqrt{2}} = \frac{18}{\frac{\sqrt{6}+\sqrt{2}}{4}}$ $\quad = 9(9\sqrt{3}-9)$

$2x = \frac{\sqrt{2}\,\cancel{(4)}\,18(\sqrt{6}-\sqrt{2})}{\cancel{4}}$ $= 81\sqrt{3} - 81$

$x = 9(\sqrt{12} - 2)$

$x = 18\sqrt{3} - 18$

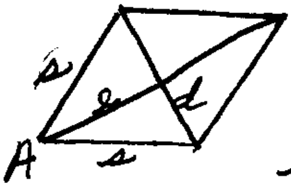
(17) $x^2 + y^2 = \frac{144}{\frac{7x^2}{x^2+y^2} + 9}$ $\quad a = 4$ A

$b = 3$

$1 = \frac{144}{7x^2 + 9x^2 + 9y^2}$ $\quad lr = \frac{2b^2}{a}$

$16x^2 + 9y^2 = 144$ $\quad = \frac{18}{4}$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$ $\quad = \frac{9}{2}$

(18) 

$$s^2 = d\,e$$

$$\sin \tfrac{1}{2}A = \frac{d}{2s} \quad \cos \tfrac{1}{2}A = \frac{e}{2s}$$

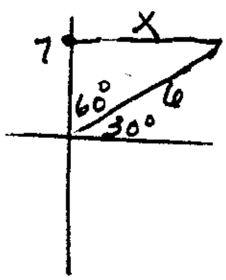
$$\sin A = 2 \sin \tfrac{1}{2}A \cos \tfrac{1}{2}A$$

$$\sin A = 2 \cdot \frac{d}{2s} \cdot \frac{e}{2s}$$

$$\sin A = \frac{2de}{4de} = \frac{1}{2}$$

$$\therefore\ A = 30^\circ$$

B

(19) 

$$x^2 = 49 + 36 - \tfrac{1}{2} \cdot 84$$

$$x^2 = 43$$

$$x = \sqrt{43}$$

A

(20)

$$\frac{\sin 100^\circ - \sin 40^\circ}{\cos 100^\circ + \cos 40^\circ} =$$

$$\frac{\sin(70^\circ + 30^\circ) - \sin(70^\circ - 30^\circ)}{\cos(70^\circ + 30^\circ) + \cos(70^\circ - 30^\circ)} =$$

$$\frac{2\cos 70^\circ \sin 30^\circ}{2\cos 70^\circ \cos 30^\circ} =$$

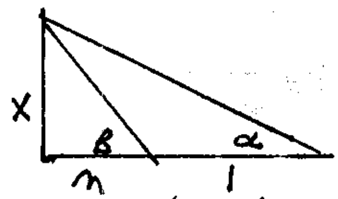
$$\tan 30^\circ = \frac{\sqrt{3}}{3}$$

B

(21) B does not fit into pattern that difference $= 72^\circ$

B

(22)



$\tan\beta = \frac{x}{n} \qquad \tan\alpha = \frac{x}{n+1}$

$$\tan\alpha = \frac{x}{\frac{x}{\tan\beta}+1}$$

A

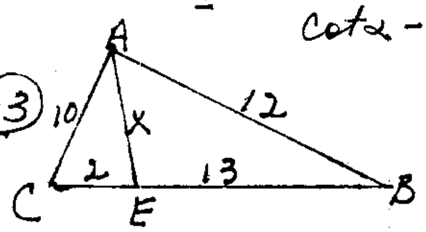
$$\tan\alpha = \frac{x\tan\beta}{x+\tan\beta}$$

$$x\tan\alpha + \tan\beta\tan\alpha = x\tan\beta$$

$$x = \frac{\tan\alpha\tan\beta}{\tan\beta-\tan\alpha}$$

$$= \frac{1}{\cot\alpha-\cot\beta}$$

(23)



A

$$144 = 100+225-300\cos C$$

$$\cos C = \frac{181}{300}$$

$$x^2 = 4+100-2\cdot 20\cdot\frac{181}{300}$$

$$x^2 = \frac{1198}{15} \qquad x^2 \sim 80$$

$$x^2 \approx \frac{1200}{15} \qquad x \sim 9$$

B

(24) $\sin^4 x = (\sin^2 x)^2$ $\qquad 3-4\cos 2x+\cos 4x$

$$= (1-\cos^2 x)^2$$

$$= \left(-\frac{\cos 2x+1}{2}\right)^2$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \frac{1-2\cos 2x+\cos^2 2x}{4}$$

$$= \frac{1-2\cos 2x+\frac{\cos 4x+1}{2}}{4}$$

25 $A + B + C = 180^\circ$

$A+B = 180 - C$

$\tan(A+B) = \tan(180 - C)$

$\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\tan A + \tan B = \tan A \tan B \tan C - \tan C$ $\underline{D}$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

Let $\tan A = A$, $\tan B = B$, $\tan C = C$

Let $D = 4^{th}$ root

$A + B + C + D = p$

$AB + AC + AD + BC + BD + CD = q$ sums of roots

$ABC + ACD + BCD + ABD = r$

$ABCD = s$

$ABC + D = p$

$ABC = p - D$

$p - D + D(AC + BC + AB) = r$

$D(AC + BC + AB - 1) = r - p$

$D = \dfrac{r - p}{AC + BC + AB - 1}$

I'd guess when I got this far!

$D = \dfrac{r - p}{q - D(ABC) - 1}$

$D = \dfrac{r - p}{q - D \cdot \frac{s}{D} - 1}$

$D = \dfrac{r - p}{q - s - 1}$