'85 Nationals

Conics Topic Test
Solutions

B 1. Center $(-1, 2)$ $h+k=1$

C 2. $-\frac{b}{2a}$ $\frac{-4}{4}$ $(-1, \ )$ $f(-1) = 2-4-7 = -9$ $(-1, -9)$

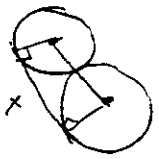
B 3. $2a = 2(8) = 16$

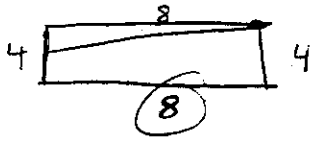
C 4. $b^2-4ac=0$ $36-4k=0$ $k=9$

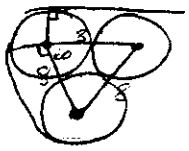
C 5. $x^2+y^2-2x+4y-3=0$ $(x^2-2x+1)+(y^2+4y+4) = 3+1+4$ $r^2=8$
$A = 8\pi$

C 6. diameter $= 6$ $A = 9\pi$

A 7.







Touches 120° of each circle

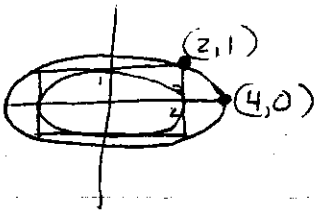
$3\times 8 + 3\left(\frac{8\pi}{3}\right) = 24+8\pi$

A 8. $y = \pm \frac{6}{7}x$ $6x+7y=0$ $6x-7y=0$

D 9. $(y-1) = \frac{6}{5}(x-2)$ $5y-5 = 6x-12$ $6x-5y=7$ III
$y-1 = -\frac{6}{5}(x-2)$ $5y-5 = -6x+12$ $6x+5y=17$ ✓ IV

A 10. $2a=6$ $a=3$ $a^2=9$ $(2,0)$ names x-axis as major.
$\frac{x^2}{9}+\frac{y^2}{b^2}=1$ $9-b^2=4$ $b^2=5$ $\frac{x^2}{9}+\frac{y^2}{5}=1$ $5x^2+9y^2=45$

D 11. $\frac{x^2}{4}+\frac{y^2}{1}=1$



$\frac{x^2}{16}+\frac{y^2}{b^2}=1$
$\frac{4}{16}+\frac{1}{b^2}=1$ $b^2=\frac{16}{12}$
$\frac{x^2}{16}+\frac{12y^2}{16}=1$ $x^2+12y^2=16$

B 12. $\frac{y^2}{36}-\frac{x^2}{b^2}=1$ $36+b^2=49$

D 13. $\frac{|3(-3)+(-1)(4)-3|}{5} = \frac{16}{5}$ $\quad x^2+6x+9+y^2+2y+1 = \frac{256}{25}$

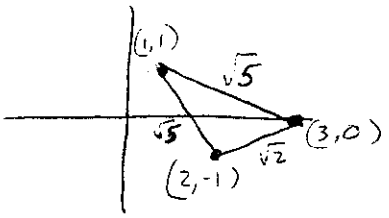
$25x^2+25y^2+150x+50y-6=0$

A 14. $3\sqrt{(x-2)^2+(y-3)^2} = \frac{2}{3}\sqrt{(x-3)^2+(y-2)^2}$

$9x^2+9y^2-36x-54y+36+81 = 4x^2+4y^2-24x-16y+36+16$

$5x^2+5y^2-12x-38y-65=0$

A 15.

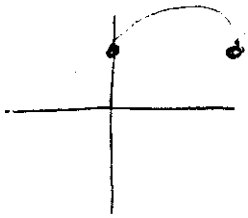


Area Triangle $\begin{matrix} 1 & 3 & 2 & 1 \\ 1 & 0 & -1 & 1 \end{matrix}$ $\quad \frac{-1-2}{2} = \frac{3}{2}$

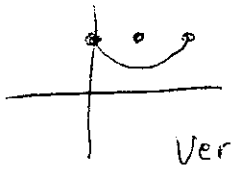
Radius $\frac{5\sqrt{2}}{4(3/2)}$ $\quad \frac{5\sqrt{2}}{6}$

$\frac{25\pi}{18} - \frac{27}{18}$

B 16.



Two possibilities $\quad \frac{1}{a} = 4 \quad a = \frac{1}{4}$ or $a = -\frac{1}{4}$



Focus = (2, 4)

Vertex = (2, 4 ± 1)

if $a = -\frac{1}{4}$

Vertex (2, 5)

$y-5 = -\frac{1}{4}(x-2)^2$

$4y-20 = -x^2+4x-4$

or

if $a = \frac{1}{4}$

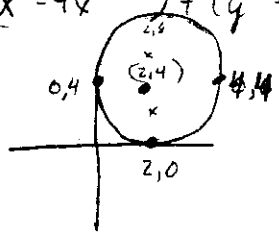
Vertex (2, 3)

$y-3 = \frac{1}{4}(x-2)^2$

$4y-12 = x^2-4x+4$

E 17. $(x^2-4x+4)+(y^2-6y+9) = -13+13$ Point

E 18.



II is false (endpoint of minor)

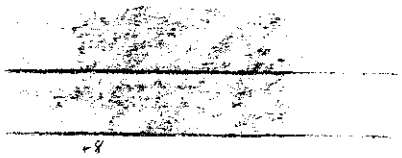
B 19. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ $\quad y = \pm\frac{3}{2}x$ $\quad 3x-2y=0$ $\quad 3x+2y=0$ I is true

(0,3) is on Long Axis IV is true

~~(±√13, 0)~~ Foci ~~False~~

(√13, 0) Foci II is True

III is false because 2a (4) is the absolute value of difference of real radii

D 20. 

D 21. $x-5 = 2(y^2-4y+4)$ $\quad (x+3) = 2(y-2)^2$

$\frac{1}{4a} = \frac{1}{8}$ $\quad (-3,2)$

directrix $x = -3\frac{1}{8}$ true

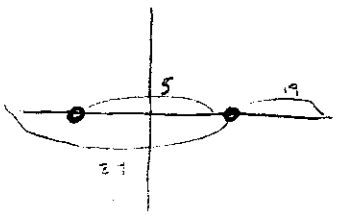
I is true

II $\frac{1}{a} = \frac{1}{2}$ True

III $(5,0)$ is True

C 22. $b^2-4ac = 9-8 = 1$ hyperbola

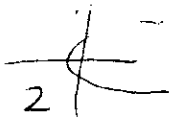
C 23. Eccentricity $= \frac{3}{2}$ hyperbola

C 24. 

$c = 2.5$

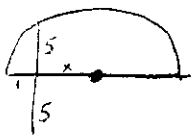
$a = 21.5$

$\frac{\frac{5}{2}}{\frac{43}{2}} = 5/43$

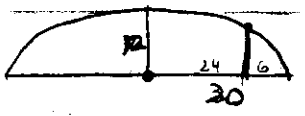


$x = \frac{1}{8}$

D 25. $\frac{y^2}{9} - \frac{x^2}{16} = 1$ foci $(0, \pm 5)$

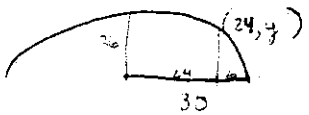
B 26. 

$x = 25$ $\quad 2r = 26$ $\quad r = 13$

D 27. 

$\frac{x^2}{400} + \frac{y^2}{144} = 1$

$\frac{x^2}{900} + \frac{y^2}{1296} = 1$



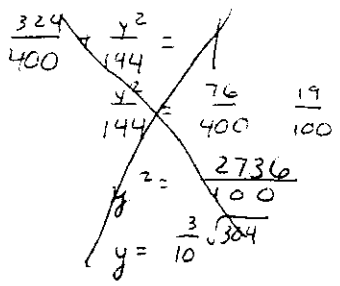
$\frac{576}{900} + \frac{y^2}{1296} = 1$

$\frac{y^2}{1296} = \frac{324}{900}$

$\frac{y}{36} = \frac{18}{30}$

$\frac{y}{36} = \frac{3}{5}$

$y = \frac{108}{5}$



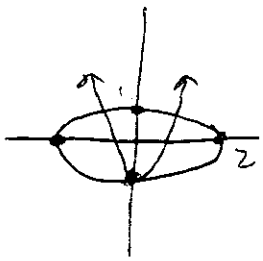
D 28. $x^2 + 4(x^2-1)^2 = 4$

$x^2 + 4(x^4 - 2x^2 + 1) = 4$

$4x^4 - 7x^2 = 0$

$x^2(4x^2 - 7) = 0$

3 points



$\frac{x^2}{4} + \frac{y^2}{1} = 1$

$y = x^2 - 1$

D 29. $\theta = 45^\circ$

$5\left(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)^2 - 6\left(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)\left(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y\right) + 5\left(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y\right)^2 = 8$

$5\left(\frac{1}{2}x^2 + xy + \frac{1}{2}y^2\right) - 6\left(\frac{1}{2}x^2 - \frac{1}{2}y^2\right) + 5\left(\frac{1}{2}x^2 - xy + \frac{1}{2}y^2\right) = 8$

$5x^2 + 5y^2 - 3x^2 + 3y^2 = 8$

$2x^2 + 8y^2 = 8$

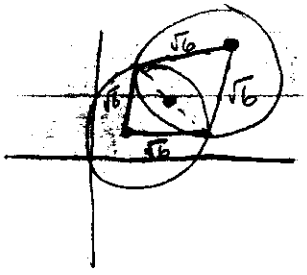
$\frac{x^2}{4} + \frac{y^2}{1} = 1$

Foci $(\sqrt{3}, 0)$ $(-\sqrt{3}, 0)$ in x' system

$x = \left(\frac{\sqrt{2}}{2}\sqrt{3} + \frac{\sqrt{2}}{2}0\right)$ $\frac{\sqrt{6}}{2}$

$y = \left(\frac{\sqrt{2}}{2}\sqrt{3} - \frac{\sqrt{2}}{2}0\right)$ $\frac{\sqrt{6}}{2}$

A 30.



$x^2 + y^2 - 2x - 2y - 4 = 0$ $(x-1)^2 + (y-1)^2 = 6$

$x^2 + y^2 - 8x - 8y + 26 = 0$ $(x-4)^2 + (y-4)^2 = 6$

Radical Axis $6x + 6y - 30 = 0$

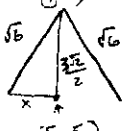
$x + y = 5$

Intersection Points

$x^2 + (5-x)^2 - 2x - 2(5-x) - 4 = 0$

$x^2 + 25 - 10x + x^2 - 2x - 10 + 2x - 4 = 0$

$2x^2 - 10x$ Doesn't help



$\left(\frac{5}{2}, \frac{5}{2}\right)$

mid pt Radical axis because $x=y$ is the line of centers

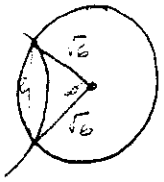
Look at Triangles

$\sqrt{\left(\frac{5}{2}-1\right)^2 + \left(\frac{5}{2}-1\right)^2}$

$\sqrt{\frac{9}{4} + \frac{9}{4}} = \frac{3\sqrt{2}}{2}$

$(\sqrt{6})^2 = \left(\frac{3\sqrt{2}}{2}\right)^2 + x^2$

$\frac{\sqrt{6}}{2} = x$ so radical axis is also $\sqrt{6}$



Area $= 2$ (Area Sector − Area Δ)

$= 2\left(\pi - \frac{3\sqrt{3}}{2}\right)$

$2\pi - 3\sqrt{3}$