Complex Numbers Test — Nationals 1996

Solutions:

B 1. $\frac{i+3}{i-2} \cdot \frac{i+2}{i+2}$

$\frac{i^2+5i+6}{i^2-4}$

$\frac{5+5i}{-5}$

$-1-i$

D 2. $3xi+12=6i(x-1)$

$3xi-6xi=-12-6i$

$-3xi=-12-6i$

$x=\frac{4}{i}+2$

$x=2-4i$

B 3. $f(1)=i,\ f(2)=3i$

$f(3)=6i-i$

$=5i$

$f(4)=10i-3i$

$=7i$

$f(5)=14i-5i$

$=9i$

$f(n)=(2n-1)i$

$f(1996)=(3992-1)i$

$=3991i$

A 4. $2(2x+y)-3(x-y)i=12-18$

$2x+y=6$

$x-y=6$

$3x=12$

$x=4,\ y=-2$

$x+y=2$

B 5. $2-i,\ 2+i$

$r_1+r_2=4$

$r_1 r_2=5$

x^2-4x+5

D 6. $(1-i)^{11}$

$[(1-i)^2]^5(1-i)$

$(1-2i+i^2)^5(1-i)$

$-32i(1-i)$

$-32-32i$

C 7. $\frac{\sqrt{-12}}{\sqrt{3}},\ \sqrt{-18}$

$\frac{2i\sqrt{3}}{\sqrt{3}} \cdot 3i\sqrt{2}$

$-6\sqrt{2}$

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A 8. $f(2i) = 2i(2i)^3 - 4(2i)^2 + 5i(2i) - 3$

$= 16i^4 - 16i^2 + 10i^2 - 3$

$= 16 + 16 - 10 - 3$

$= 19$

C 9. $-3 + \sqrt{3} = 2\sqrt{3}\ \text{Cis}\ 150°$

$r = \sqrt{12}$

$= 2\sqrt{3}$

$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) + 180°$

$= 150°$

B 10. $\sum_1^{10} (2i)^n = 2i + (2i)^2 + \ldots + (2i)^{10}$

$2i + 8i^3 + 32i^5 + 128i^7 + 512i^9$

$410i$

$4i^2 + 16i^4 + 64i^6 + 256i^8 + 1024i^{10}$

$-820 + 410i$

B 11. $x^3 - 8x^2 + 25x - 26 = 0$

2	1	−8	25	−26
		−2	−12	26
	1	−6	13	0

$x^2 - 6x + 13 = 0$

$x = 2, 3 \pm 2i$

D 12. $z = a + bi,\ \bar{z} = a - bi$

$z\bar{z} = a^2 - b^2 i^2$

$= a^2 + b^2$ real no.

$z + \bar{z} = 2a$ real no.

B 13. $x^2 = i$

$x = \pm\sqrt{i}$

$\pm\sqrt{i} = a + bi$

$i = a^2 - b^2 + 2abi$

$a^2 - b^2 = 0,\ 2ab = 1$

$a = \frac{1}{2b}$

$\frac{1}{4b^2} - b^2 = 0$

$1 - 4b^4 = 0$

$b^4 = \frac{1}{4}$

$b^2 = \pm\frac{1}{2}$

$b = \pm\frac{\sqrt{2}}{2},\ a = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i,\ -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

D 14. $i^{123} - i^{39} + i^{-3} - i$

$-i + i + \frac{1}{i^3} - i$

$0 + i - i$

0

E

15. $\frac{(1+i)^{-12}(1-\sqrt{3})^5}{[(1+i)^2]^6}$

$\frac{(1+2i+i^2)^6}{(64i)^{-1}}$

$\left(\frac{1}{64i}\right)(2^5\ \text{Cis}(-300))$

$-\frac{1}{64}i \cdot 32\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$\frac{\sqrt{3}}{2} - \frac{1}{4}i$

$r = \sqrt{4}$

$= 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1}$

$\theta = -60$

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B 16. $e^{i2x} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$0 \le x \le 2\pi$

$Re^{i\theta} = R\cos\theta$

$\cos 2x + i\sin 2x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$2x = \cos^{-1}\frac{\sqrt{2}}{2}$ $\quad 2x = \sin^{-1} -\frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4}, \frac{7\pi}{4}$ $\quad 2x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = \frac{\pi}{8}, \frac{7\pi}{8}$ $\quad x = \frac{5\pi}{8}, \frac{7\pi}{8}$

$x = \frac{7\pi}{8}$

C 17. $|5-12i| = \sqrt{5^2+12^2}$

$= \sqrt{169}$

$= 13$

~~A~~ E 18. $3x^2 - 4kx + 1 = 0$

$b^2 - 4ac < 0$

$16k^2 - 12 < 0$

$4k^2 - 3 < 0$

$-\frac{\sqrt{3}}{2} < k < \frac{\sqrt{3}}{2}$

$(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2})$

C 19. $x = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, $y = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$

$x^2 = \frac{1}{4} + i + \frac{1}{4}i^2$ $\quad y^2 = \frac{1}{4} - i + \frac{1}{4}i^2$

$= i$ $\quad = -i$

$x^4 = i^2$ $\quad y^4 = i^2$

$= -1$ $\quad = -1$

$x^5 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ $\quad y^5 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$x^5 + y^5 = -\sqrt{2}$

B 20. $\frac{2-i}{4+i} - \frac{2i}{3-i} = a+bi$

$\frac{(2-i)(3-i) - 2i(4+i)}{12 - i - i^2} =$

$\frac{6 - 5i + i^2 - 8i - 2i^2}{13 - i} =$

$\frac{7 - 13i}{13+i} \cdot \frac{13+i}{13+i} =$

$\frac{52}{85} - \frac{81}{85}i$

$a+b = -\frac{29}{85}$

C 21. $2+3i$, $4-4i$

distance $= \sqrt{2^2+7^2}$

$= \sqrt{53}$

C 22.

D 23. $3-3i = 3\sqrt{2}\cos(-45°)$

$r = \sqrt{9+9}$

$= 3\sqrt{2}$

$\theta = \tan^{-1}(-1)$

$= -45°$

$765° - 720° = 45°$

and $3\sqrt{2}\cos 765 = 3+3i$

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C 24. $(16\,\mathrm{cis}\,120^\circ)^{1/4}$

$= 16^{1/4}\,\mathrm{cis}\left(\frac{120+360k}{4}\right)$

$= 2\,\mathrm{cis}(30+90k)$

$x_1 = \sqrt{3} + i$

$x_2 = -1 + \sqrt{3}i$

$x_3 = -\sqrt{3} - i$

$x_4 = 1 - \sqrt{3}i$

A 25. $2(\cos 4 + i \sin 4)$

$re^{i\theta} = r(\cos\theta + i\sin\theta)$

$= 2(\cos 4 + i \sin 4)$

$2e^{4i} = 2(\cos 4 + i \sin 4)$

D 26. $2(\cos\alpha + i\sin\alpha)(\cos\alpha - i\sin\alpha)$

$2(\cos^2\alpha - i^2\sin^2\alpha)$

$2(\cos^2\alpha + \sin^2\alpha)$

2

D 27. $ix = 1$

$x = \frac{1}{i}$

$x = -i$

C 28. $x^3 = -1$

$x^3 + 1 = 0$

$(x+1)(x^2 - x + 1) = 0$

$x = -1,\ x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$A = \frac{1}{2}\left(\frac{3}{2}\right)(\sqrt{3})$

D 29. Square

B 30. $3ix^2 - 11x - 6i$

$-3i$	$3ix - 11 - 6i$
	$9 \quad 6i$
$3ix - 2$	0

$(x + 3i)(3ix - 2)$

Tie-Breaker

B 1. $z = 2+3i,\ \bar{z} = 2-3i$

$z\bar{z} = 13$

$2z = 4 + 6i$

$-\bar{z}^2 = 5 + 12i$

$z\bar{z} + 2z - z^2 = 22 + 18i$

C 2. $1 - \sqrt{2}i,\ 1 + \sqrt{2}i,\ 2+3i,\ 2-3i$

$r_1 + r_2 = 2 \quad r_1 + r_2 = 4$

$r_1 \cdot r_2 = 3 \quad R_1 R_2 = 13$

$(x^2 - 2x + 3)(x^2 - 4x + 13) = 0$